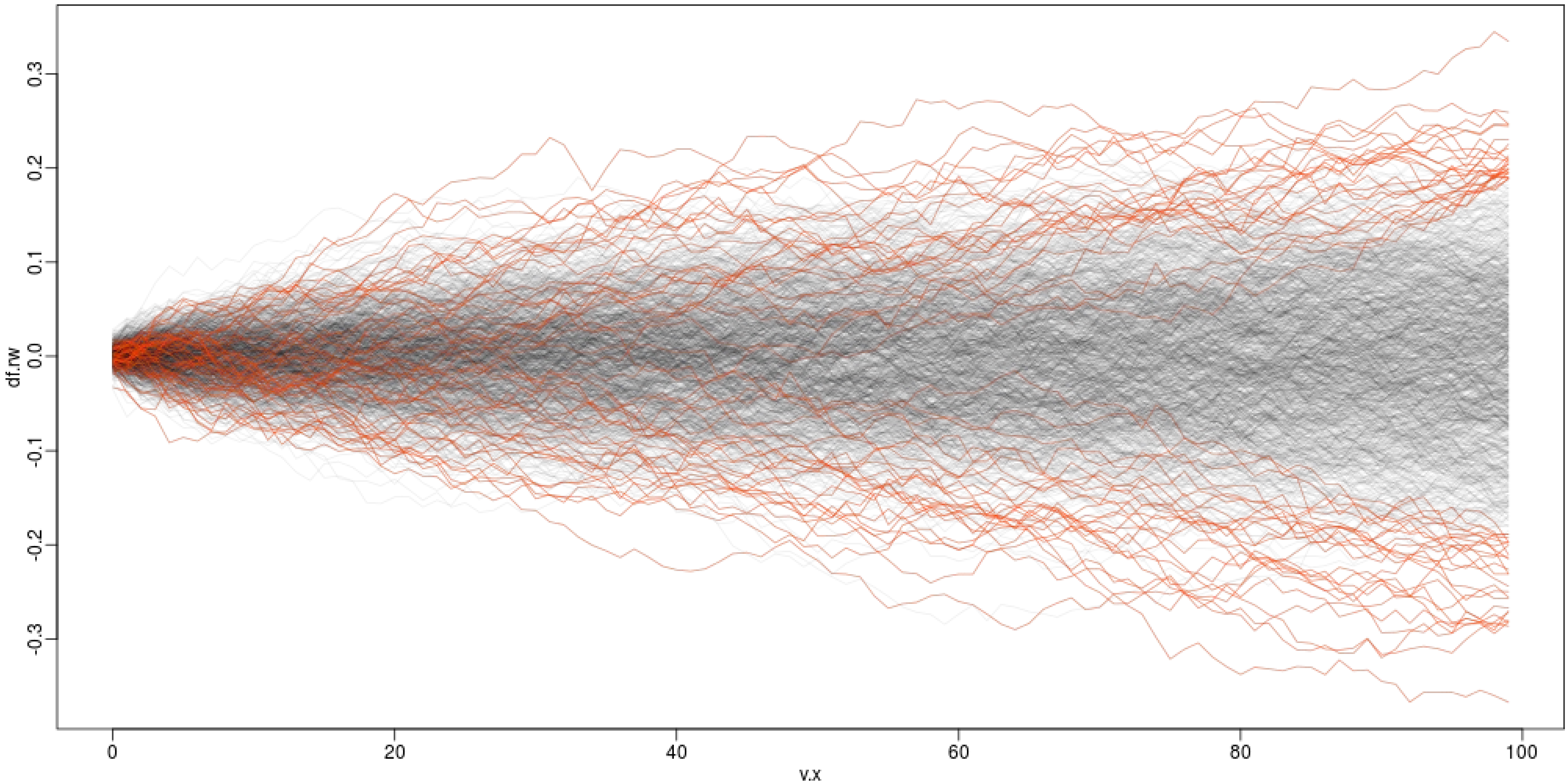
An introduction to time-series models for ecological data

久保拓弥 mailto:kubo@ees.hokudai.ac.jp



Do not apply simle GLMs

to time-series (TS) data!

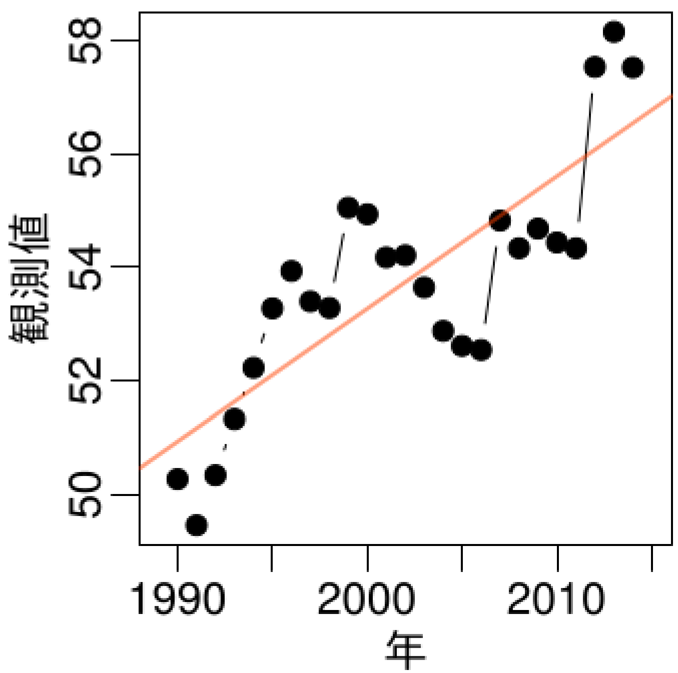
However, hierarchical Bayesian

models are

still effective to TS data

(Bad model 1) fit GLM to TS

NO!



「ゆーいな傾き」を
ねつぞうする原因

傾きの検定やめて
AIC モデル選択
しても同様になる

検定とかモデル選択とかそういう問題ではない

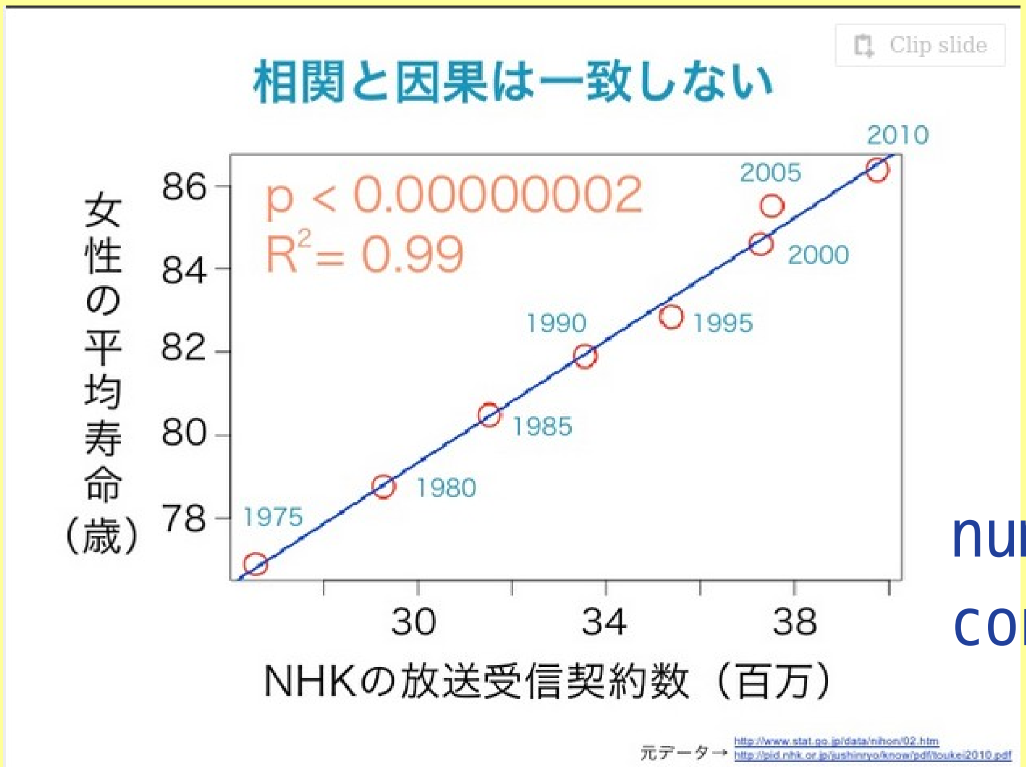
統計モデルがおかしい?

(Bad model 2) TS $Y_t \sim$ TS X_t

so called

“spurious regression”

women longevity



number of NHK contracts

http://www.slideshare.net/takehikoihayashi/ss-13441401

Statistical modeling for time series (TS) data

- Do NOT fit GLM to TS data
- A basic component: Random walk (RW) model
- RW + GLM → State Space Model (SSM)

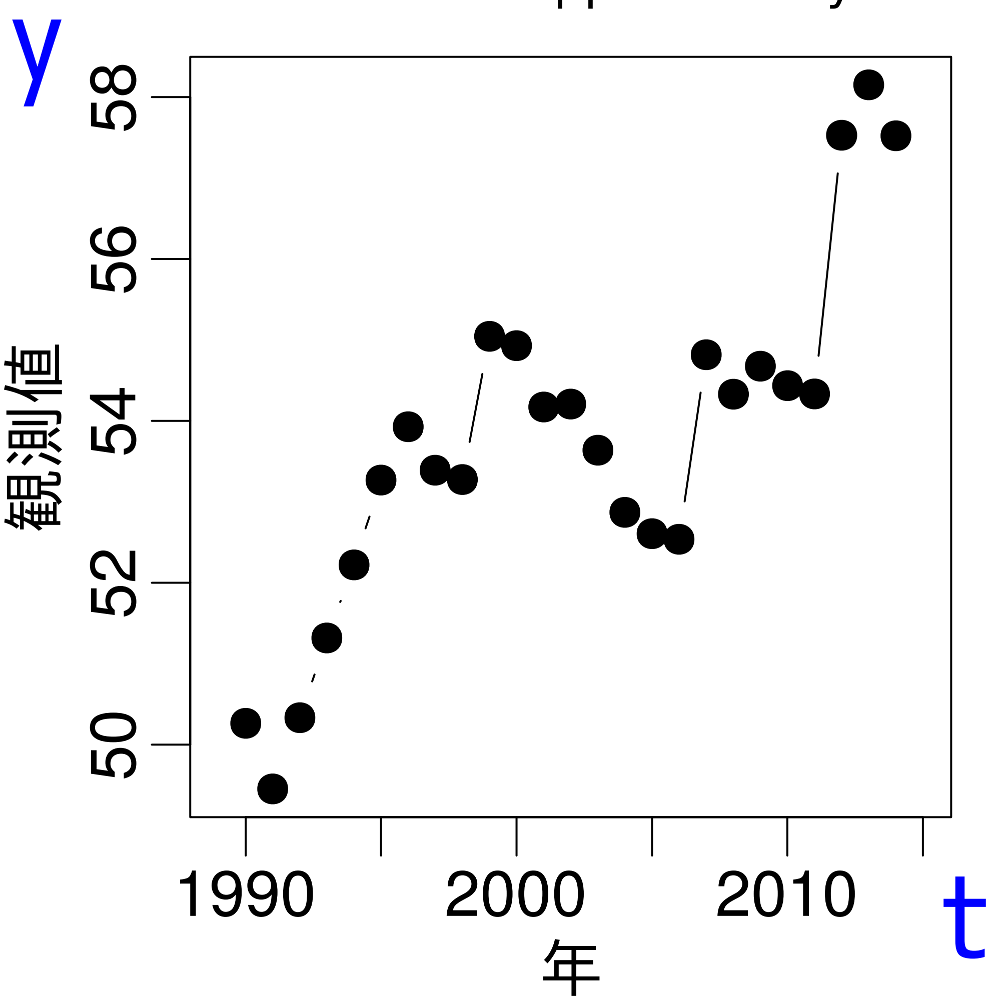
状態空間モデル

simple GLM: A bad model

for time-series (TS) data

このような時系列データがあったとしましょう

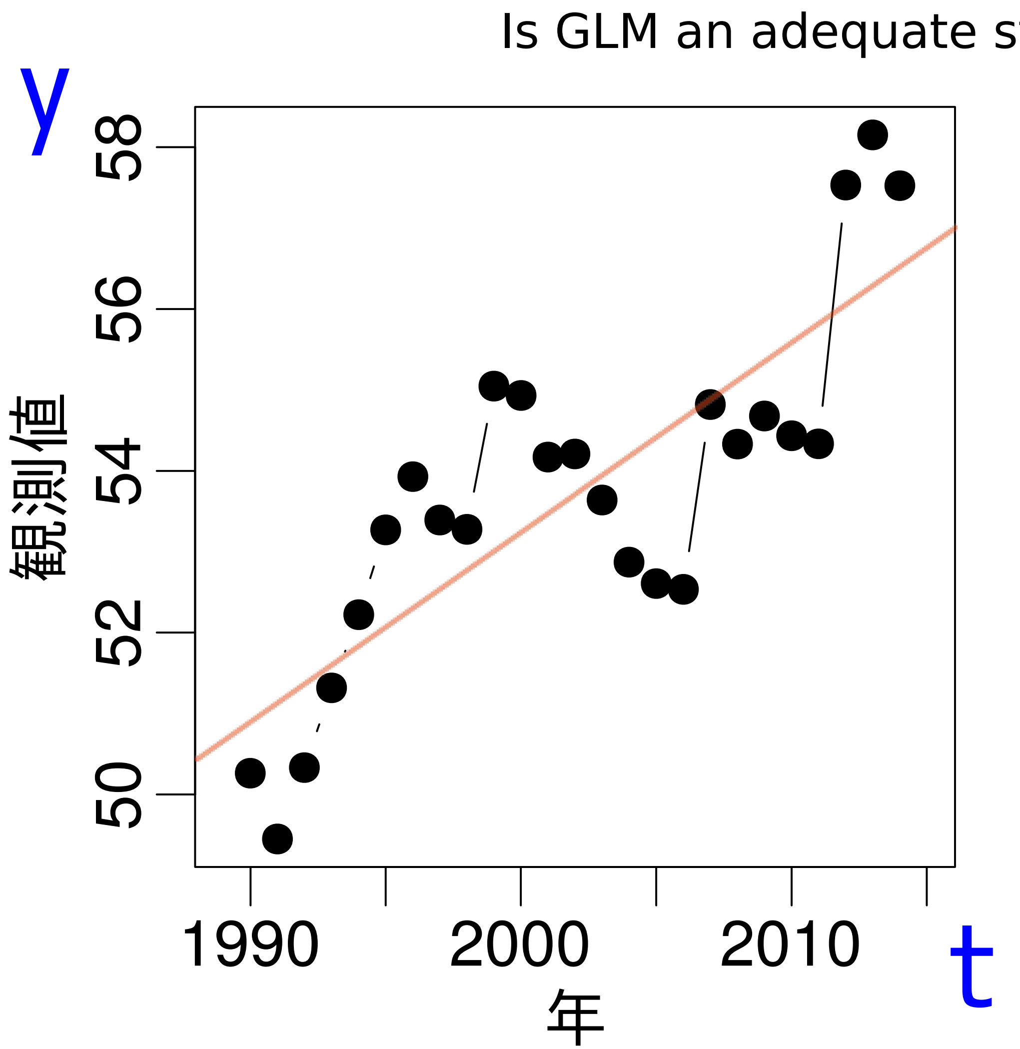
Suppose that you have a time-series data



y は何か連続値としましょう

y: some continuous measurements

時系列データの統計モデリング入門

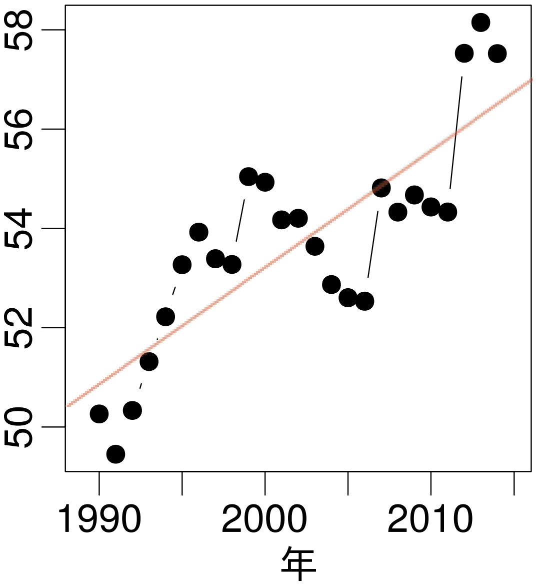


glm(y ~ t)

…とモデル
　をあてはめてみた

「やったーゆーいだ!!」……??

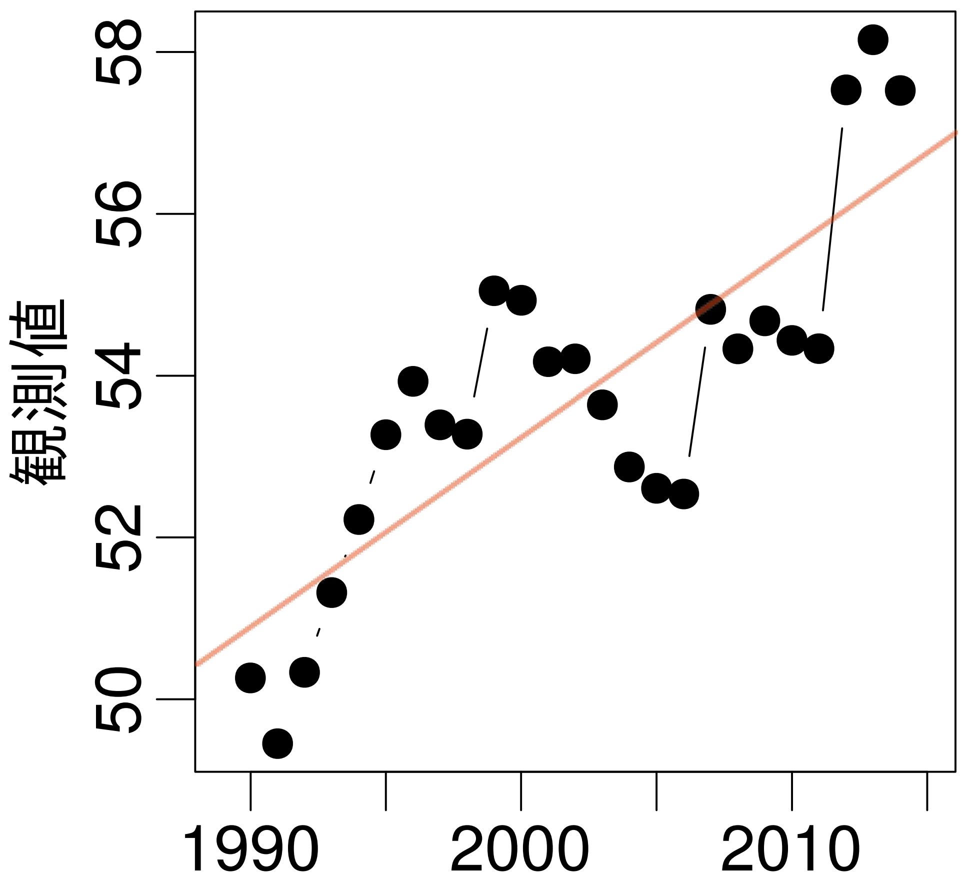
Significant ??? No!




```
> summary(glm(formula = y ~ t))

Deviance Residuals:
    Min        1Q    Median        3Q       Max
-2.1295   -1.0583  -0.0817    0.9860    2.0188

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -414.5655    71.4761   -5.80  6.6e-06
t              0.2339     0.0357    6.55  1.1e-06
```

A bad modeling: glm(時系列Y ~ 時間 t)

時系列の各点は独立ではない



each data points are

affective by previous

data point!

NOT independents!

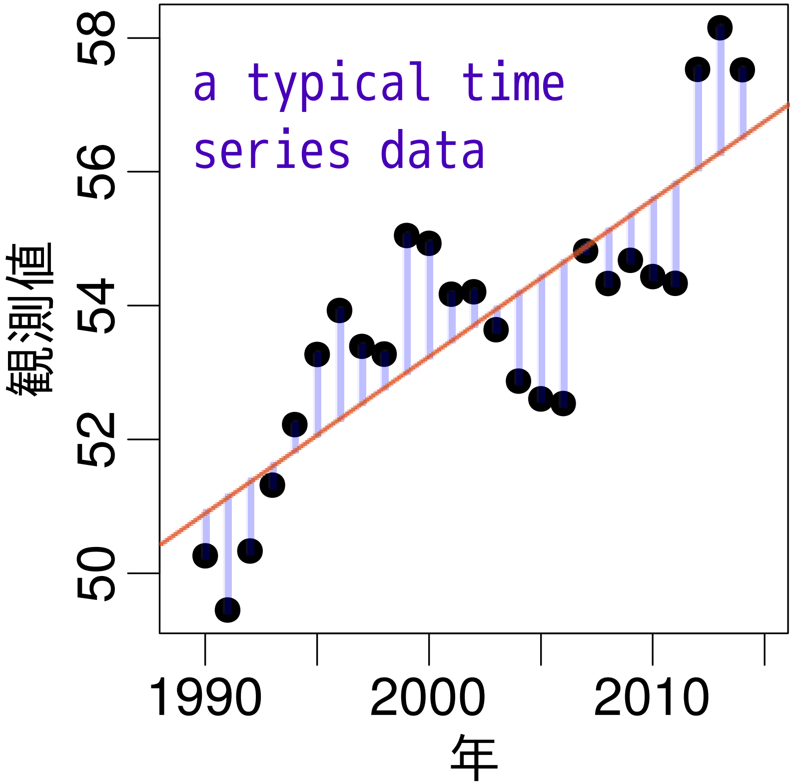
「ゆーい」な傾き

a fake significancy!

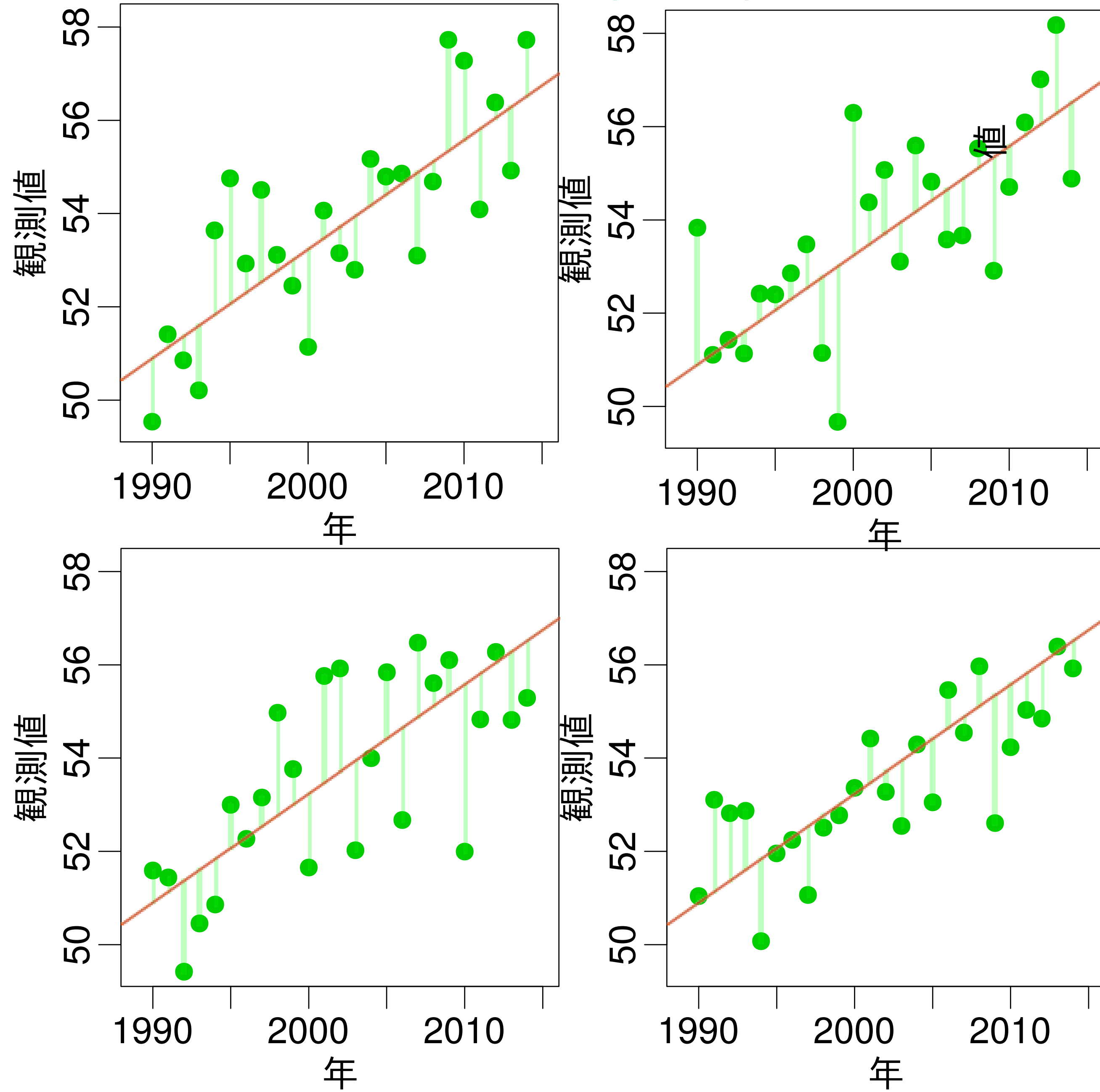
検定とかモデル選択とかそういう問題ではない

統計モデルがおかしい?

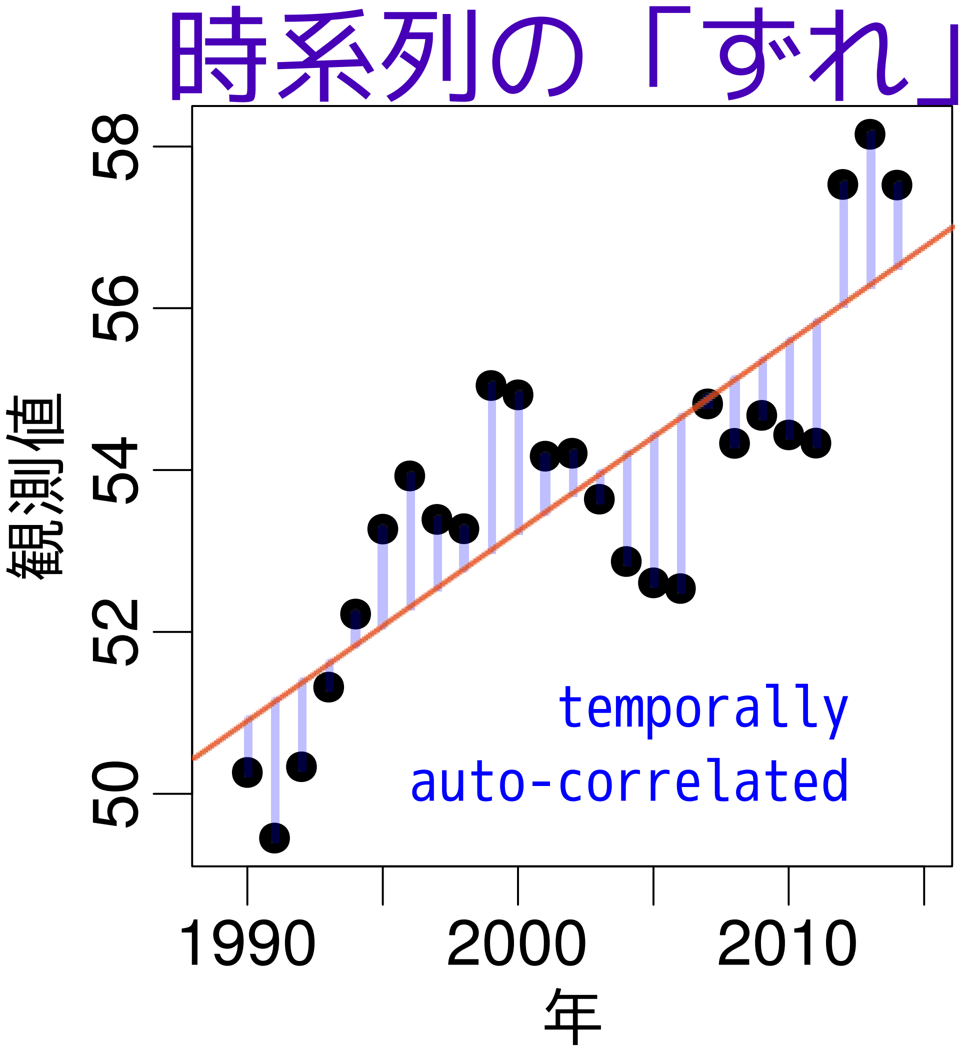
時系列の「ずれ」　GLM のずれ

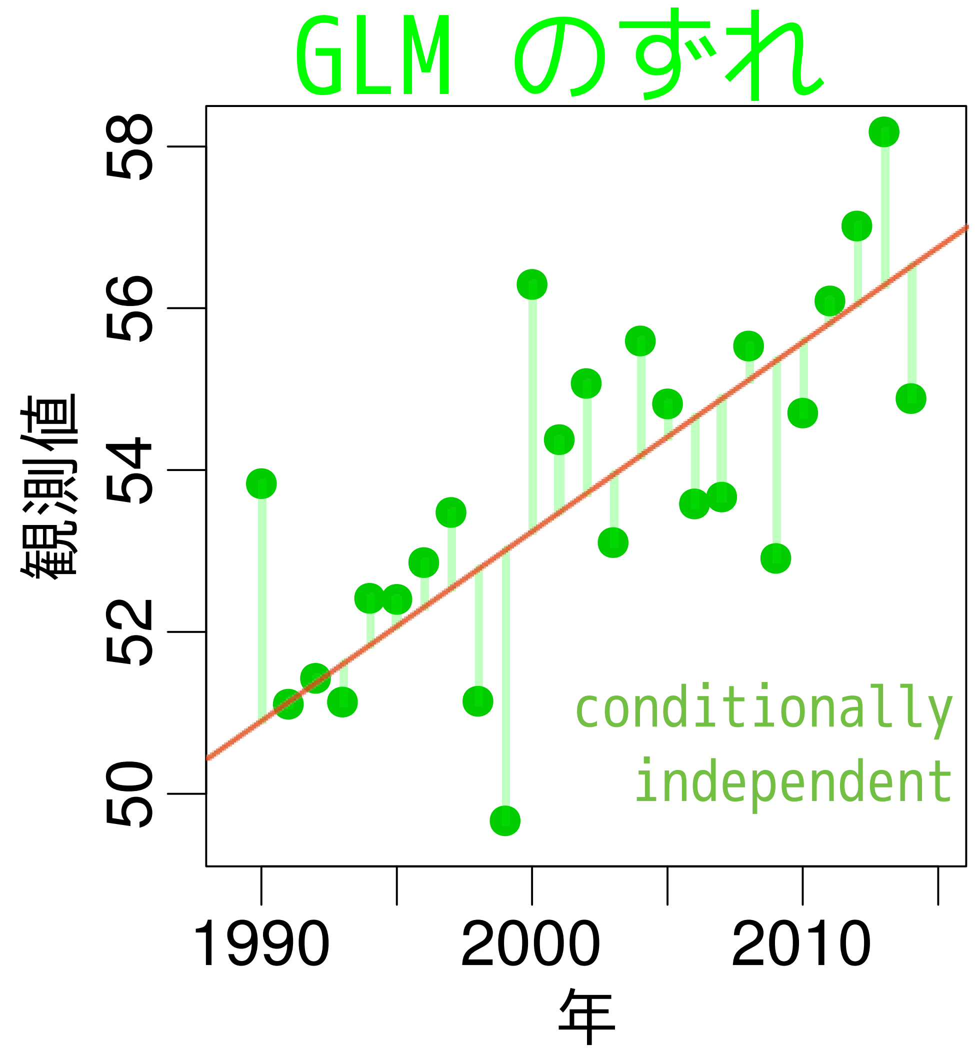


conditionally independent



can you see the differences?

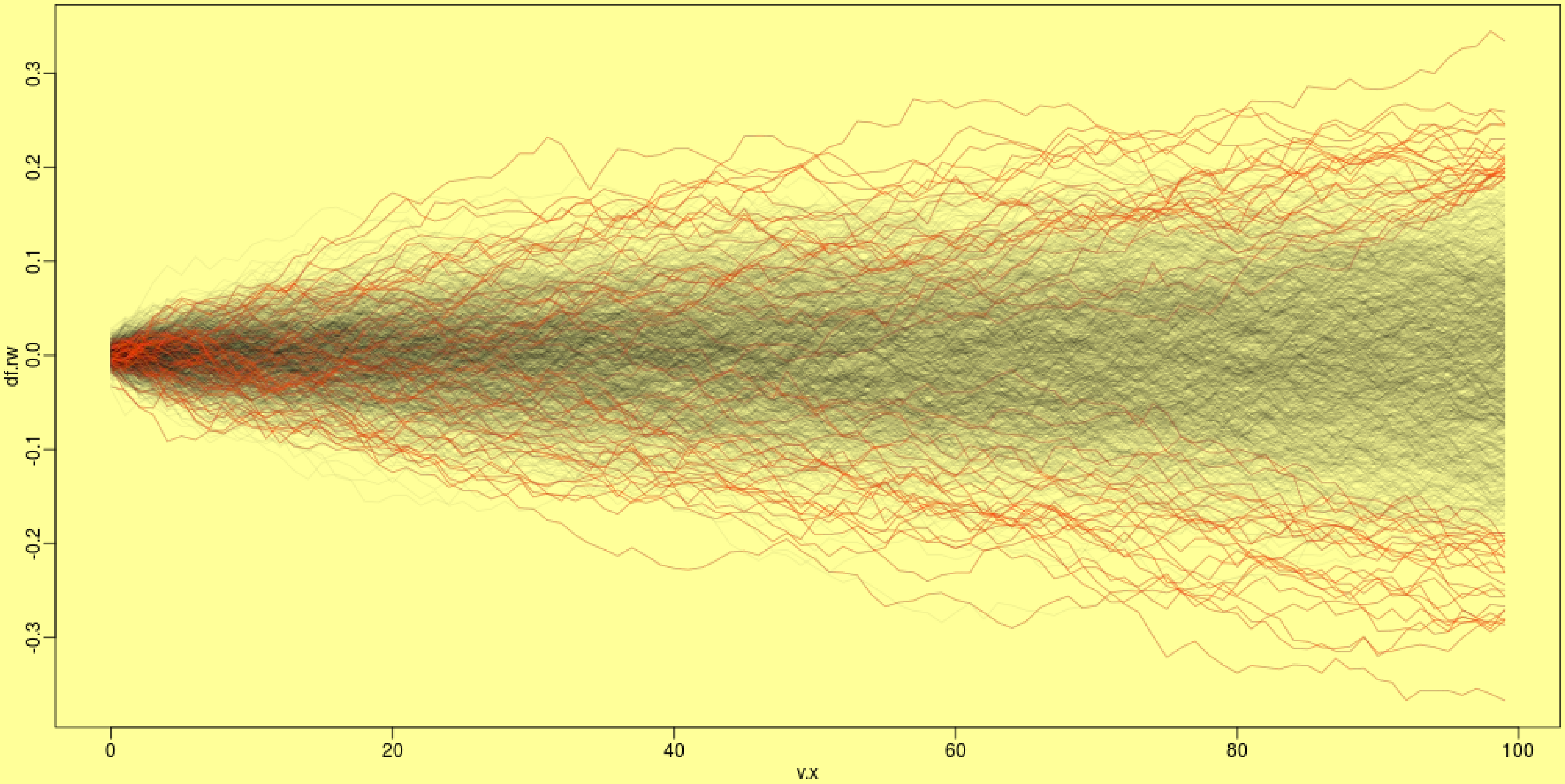




See time auto-correlation!

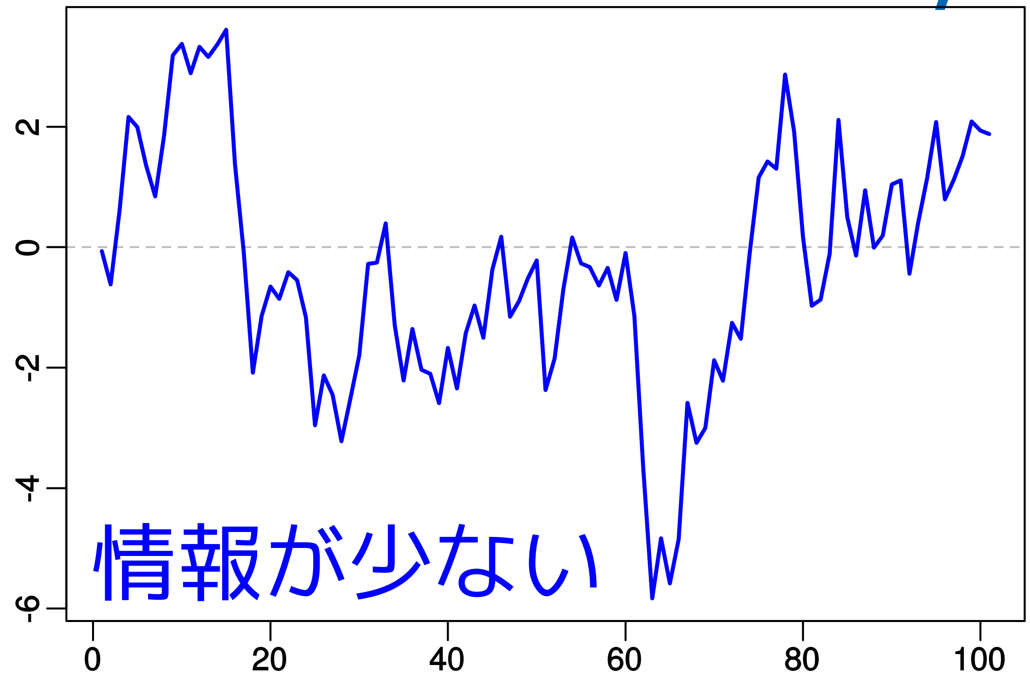
時間的自己相関がある　　時間的自己相関がない

Time series model as Random walk (乱歩)

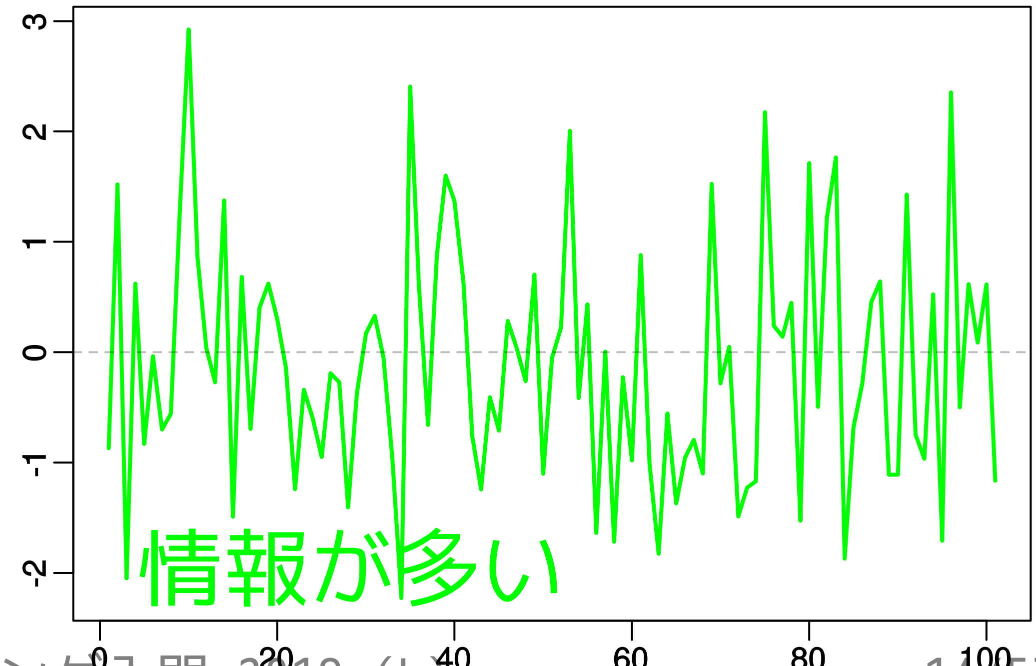


non-stationary

Depending on previous data point



Independent at each time



temporal auto-correlation function (ACF)

(略称:自己相関, 時間相関)

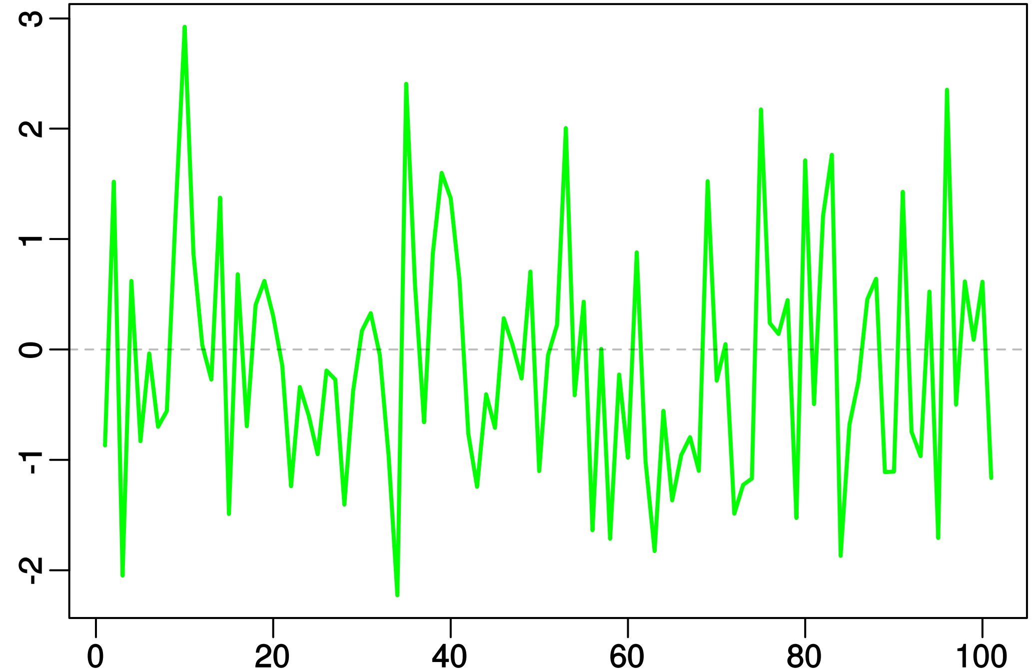
を調べたらいいの?

$$\rho_k = \frac{\mathrm{Cov}(y_t, y_{t-k})}{\sqrt{\mathrm{Var}(y_t) \cdot \mathrm{Var}(y_{t-1})}}$$

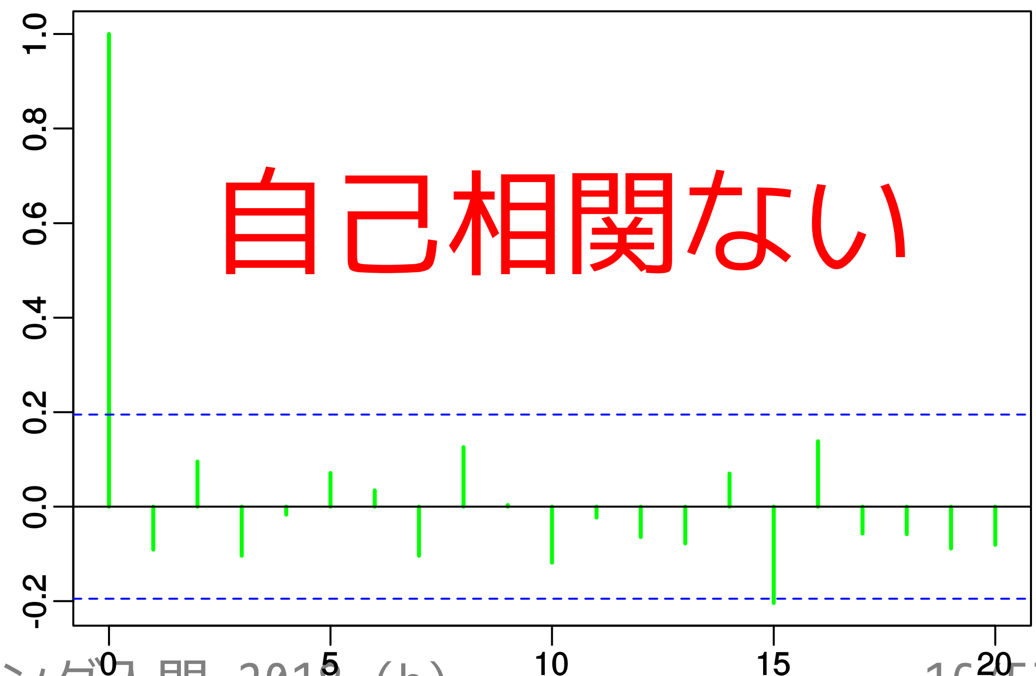
R の ts クラス: 時系列をあつかう

```
plot(ts(Y))
```

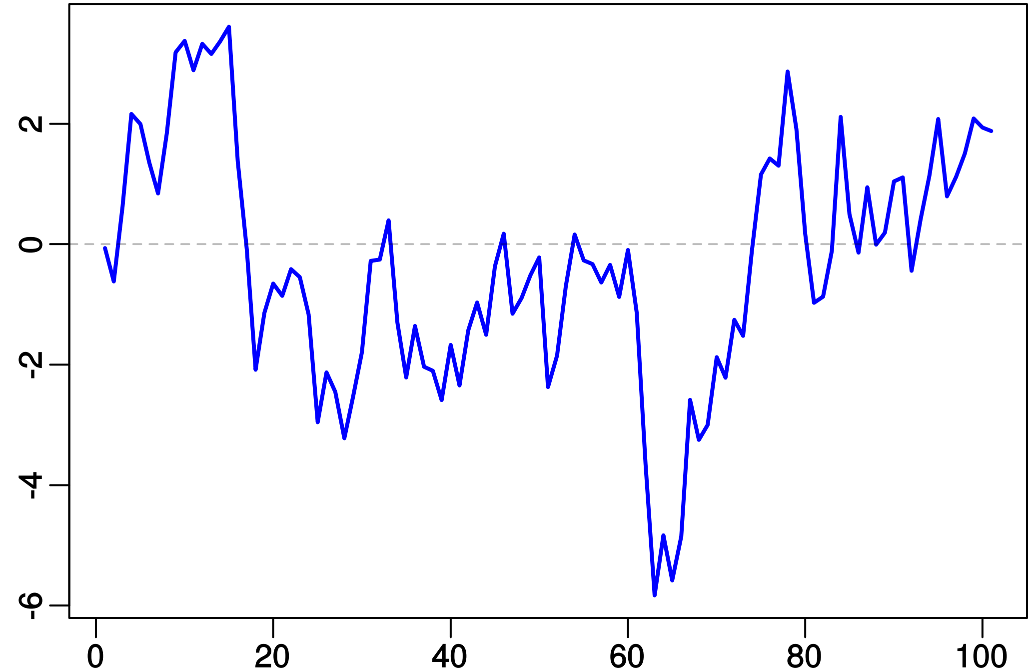
これはたんなる
100 個の正規乱数




```
plot(acf(ts(Y)))
```

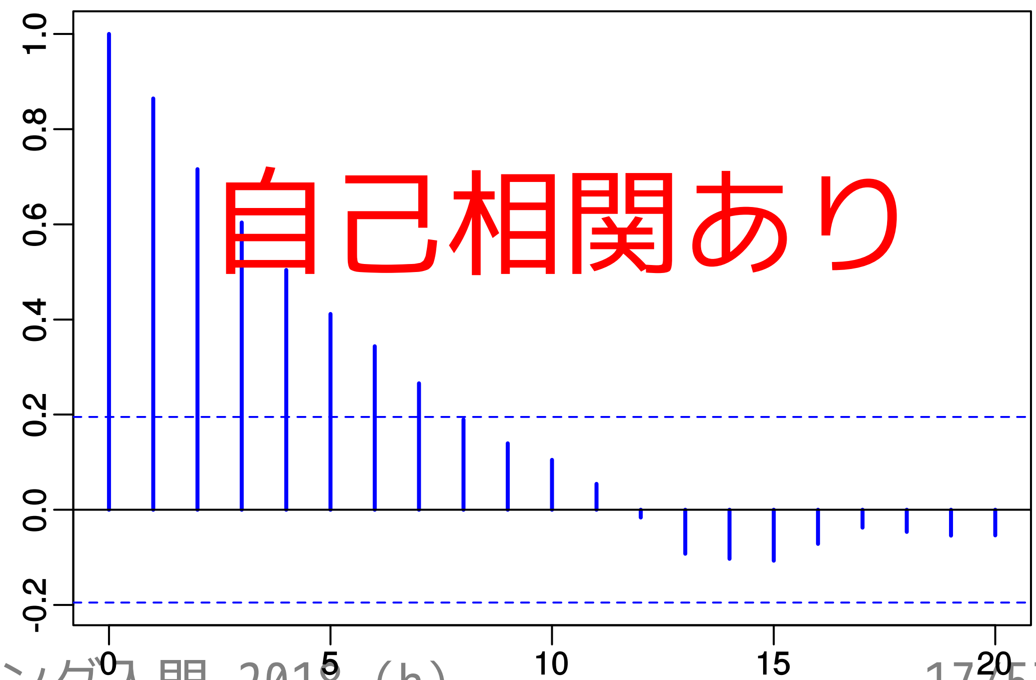


plot(ts(Y))



plot(acf(ts(Y)))

自己相関減衰



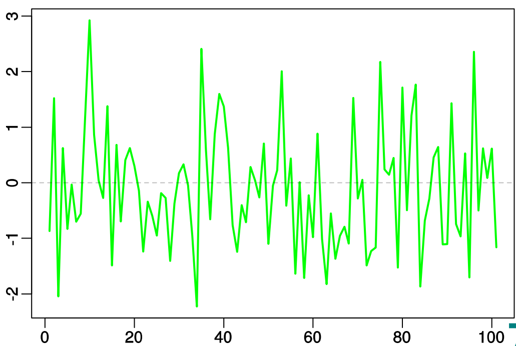
時間的自己相関

いつも役にたつわけではない?

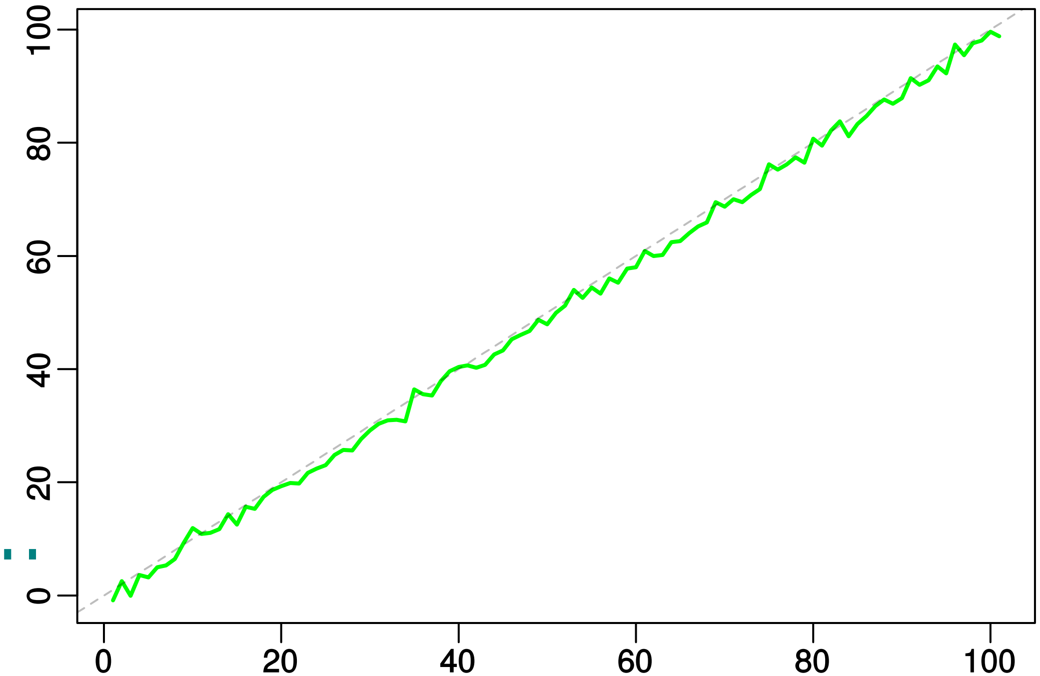
$$\rho_k = \frac{\mathrm{Cov}(y_t, y_{t-k})}{\sqrt{\mathrm{Var}(y_t) \cdot \mathrm{Var}(y_{t-1})}}$$

各点独立のデータをナナメにすると?

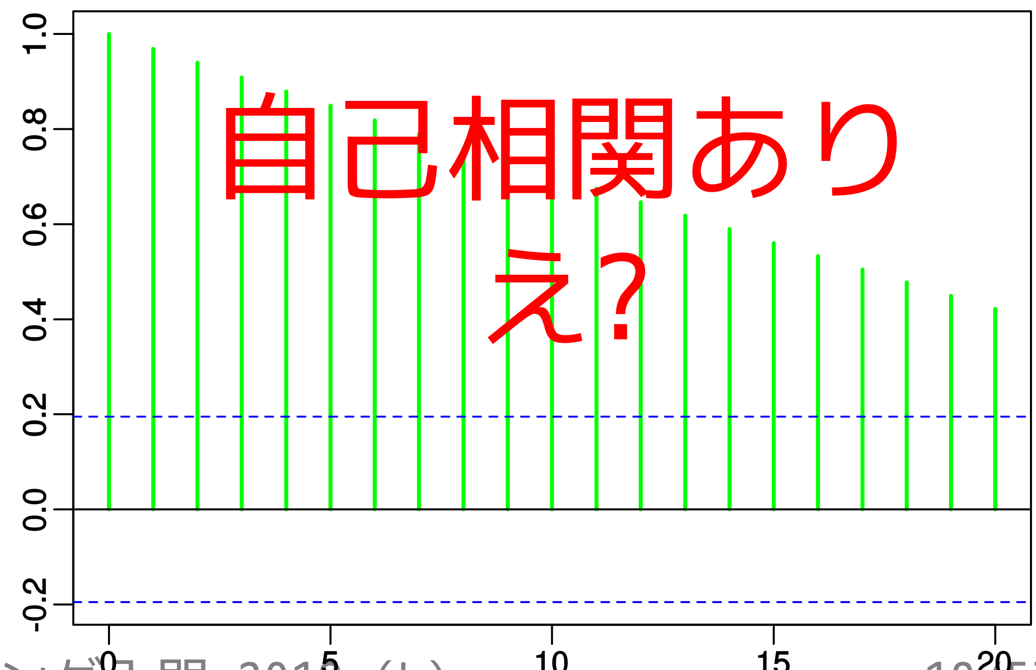
plot(ts(Y))



これを
ナナメに
したもの
なんだけど…



plot(acf(ts(Y)))



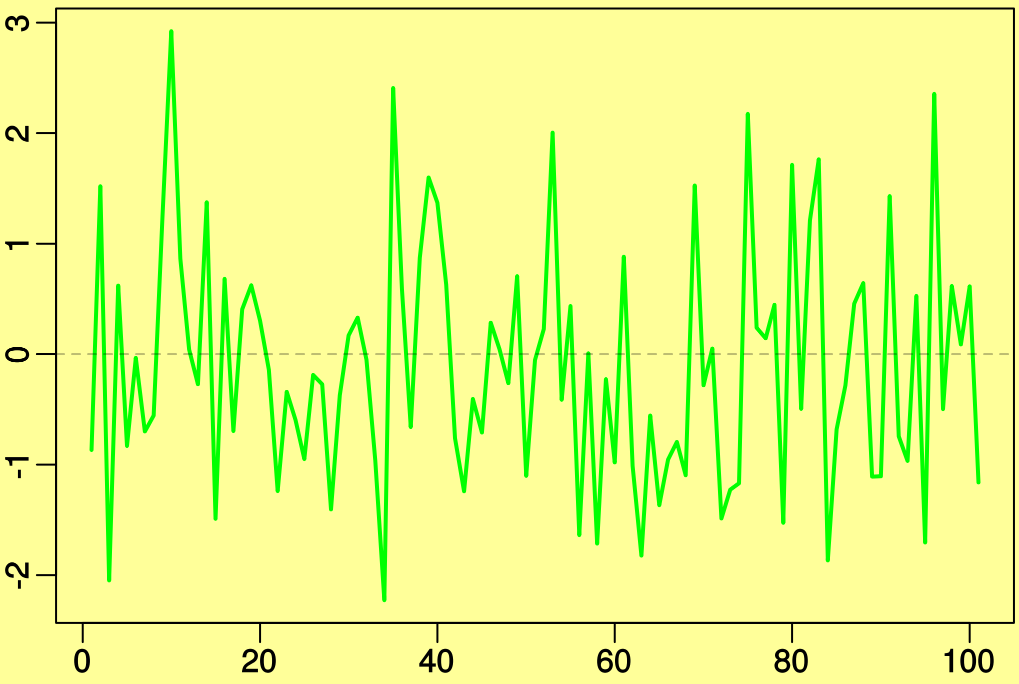
自己相関あり
え?

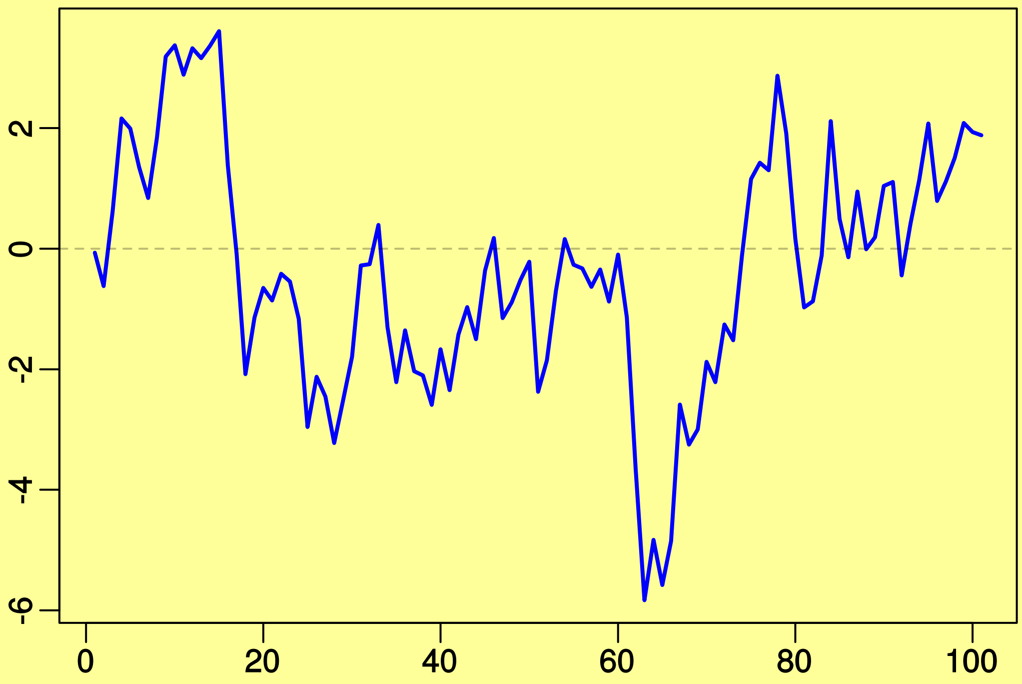
状態空間モデルでたちむかう

時系列データ解析

The State-space model,

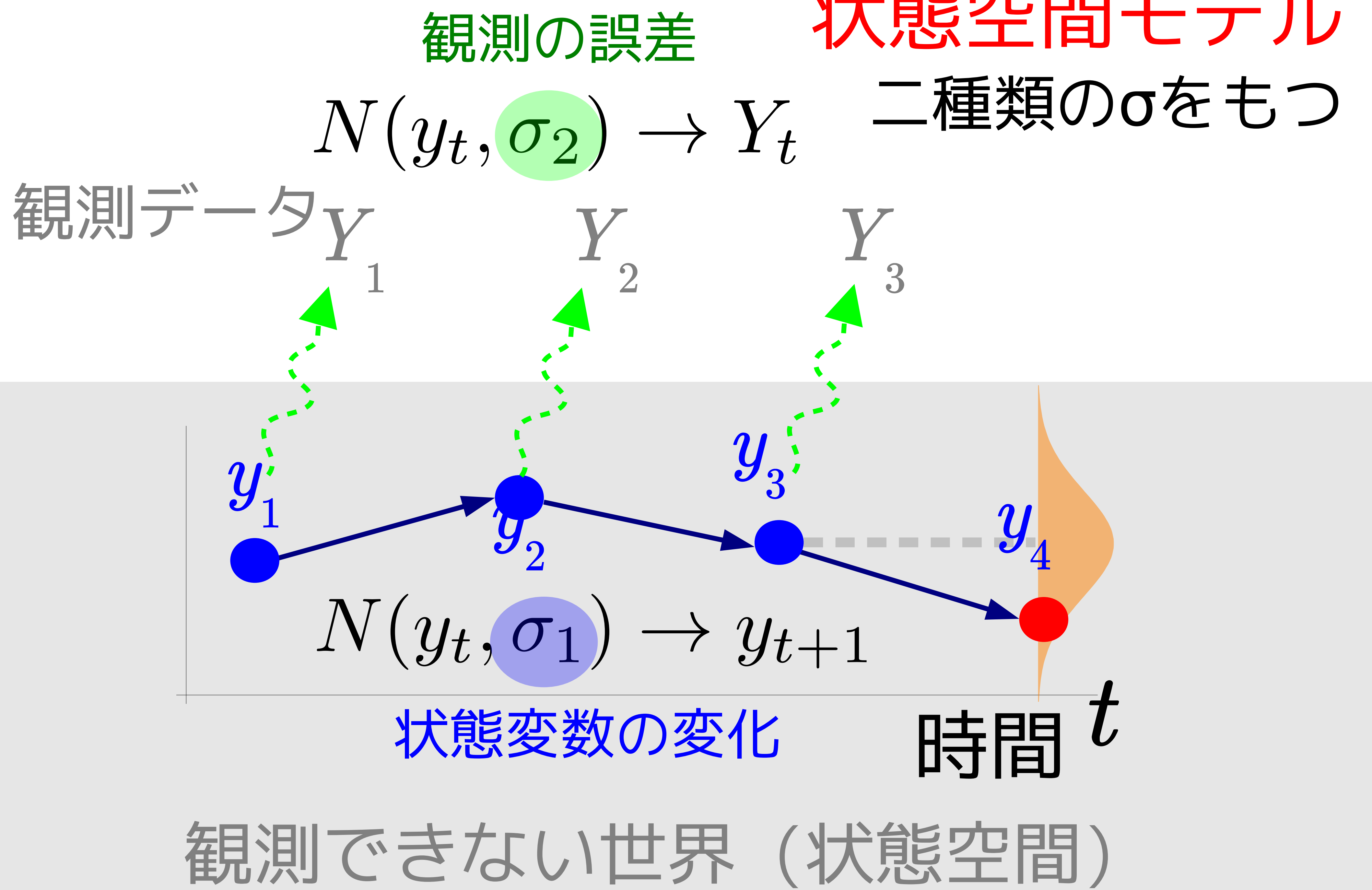
a unified time-series model





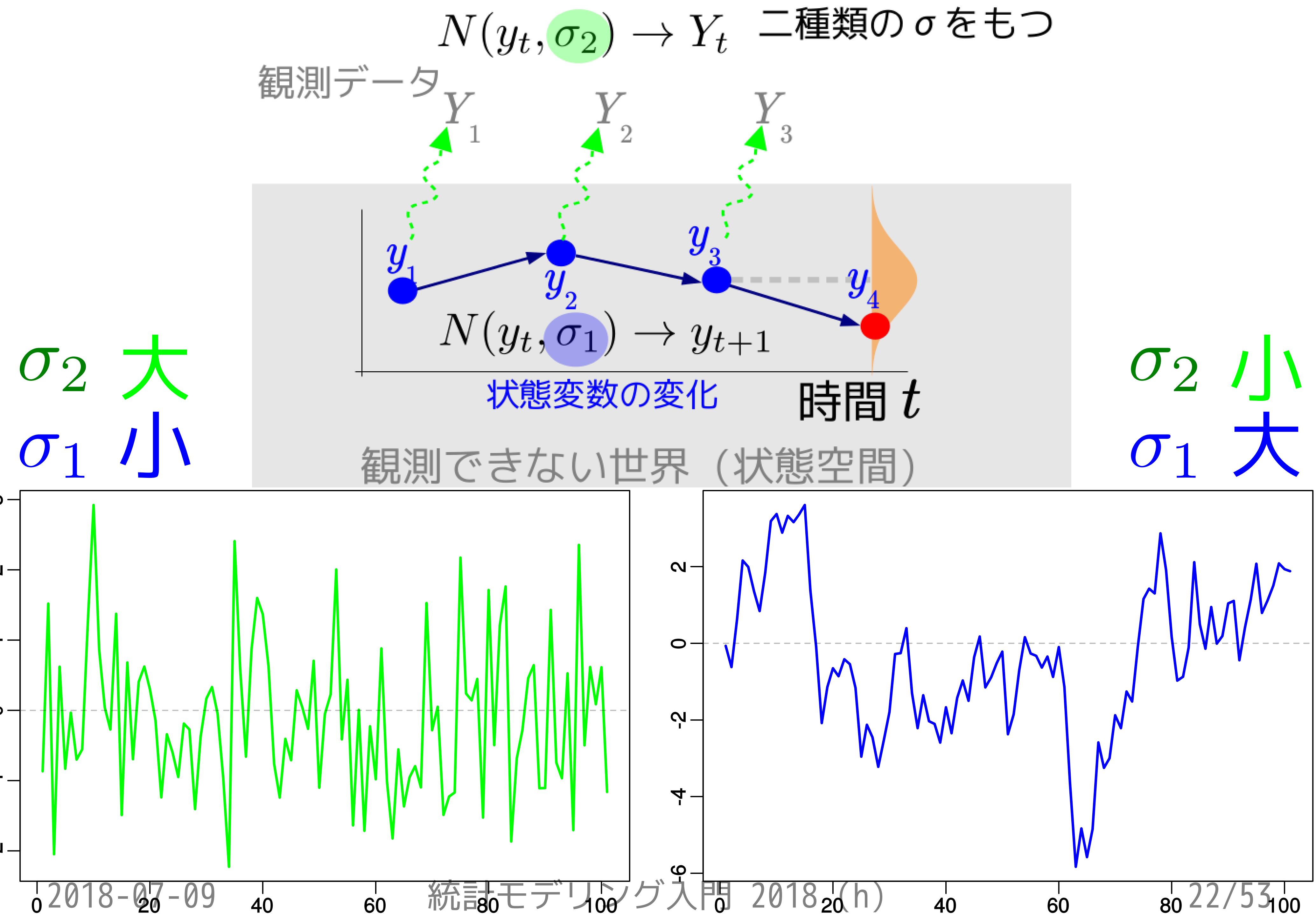
状態空間モデル

二種類のσをもつ



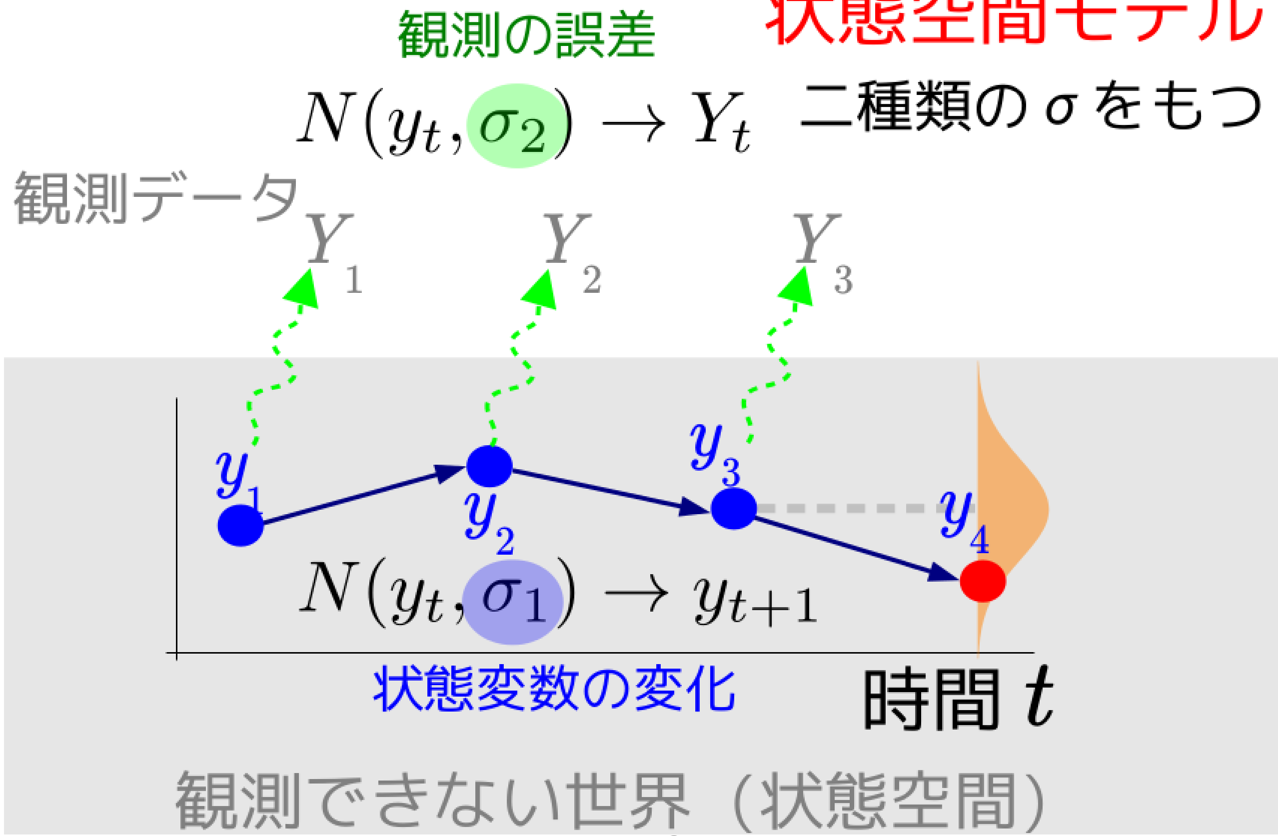
状態空間モデル

二種類の σ をもつ



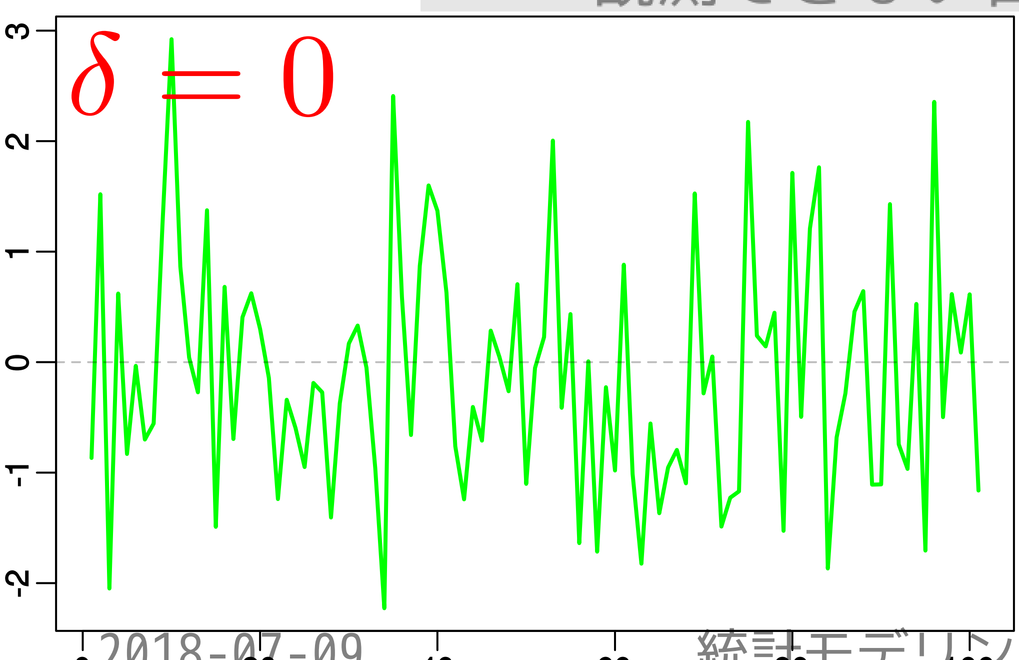
状態空間モデル

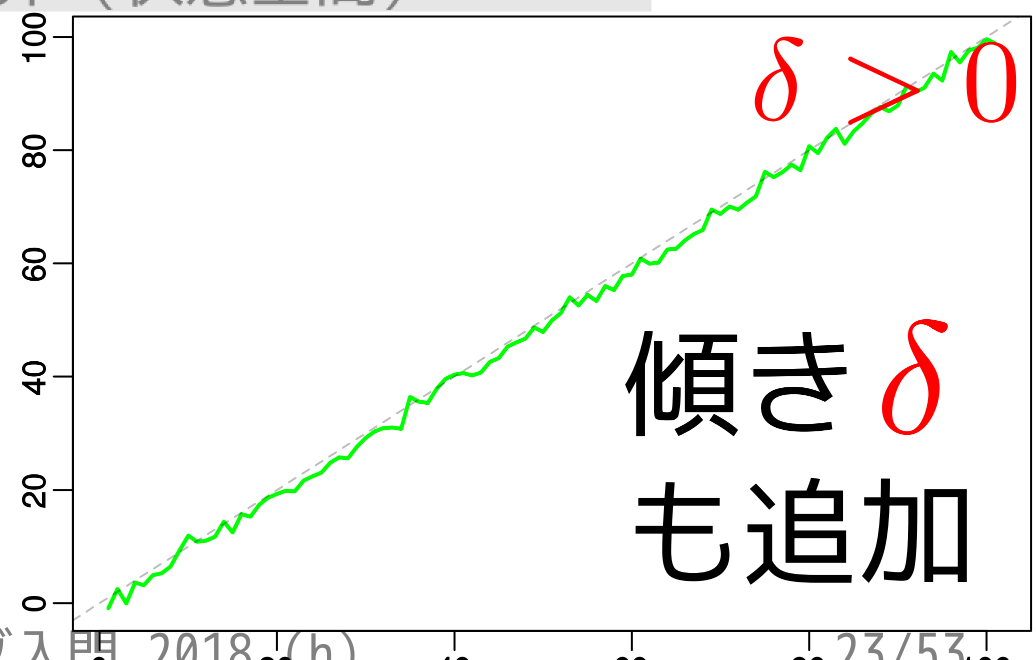
二種類の σ をもつ



σ_2 大

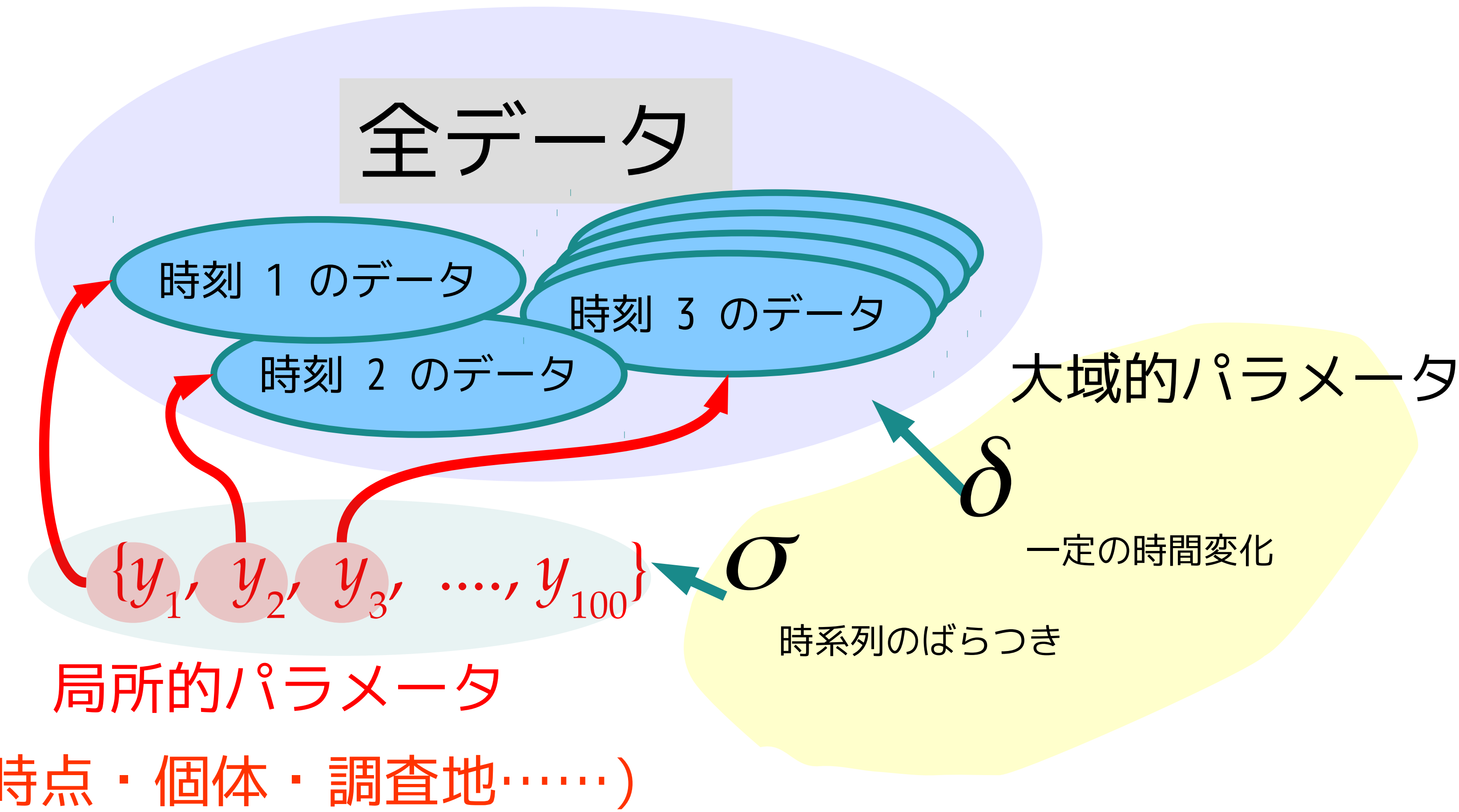
σ_1 小



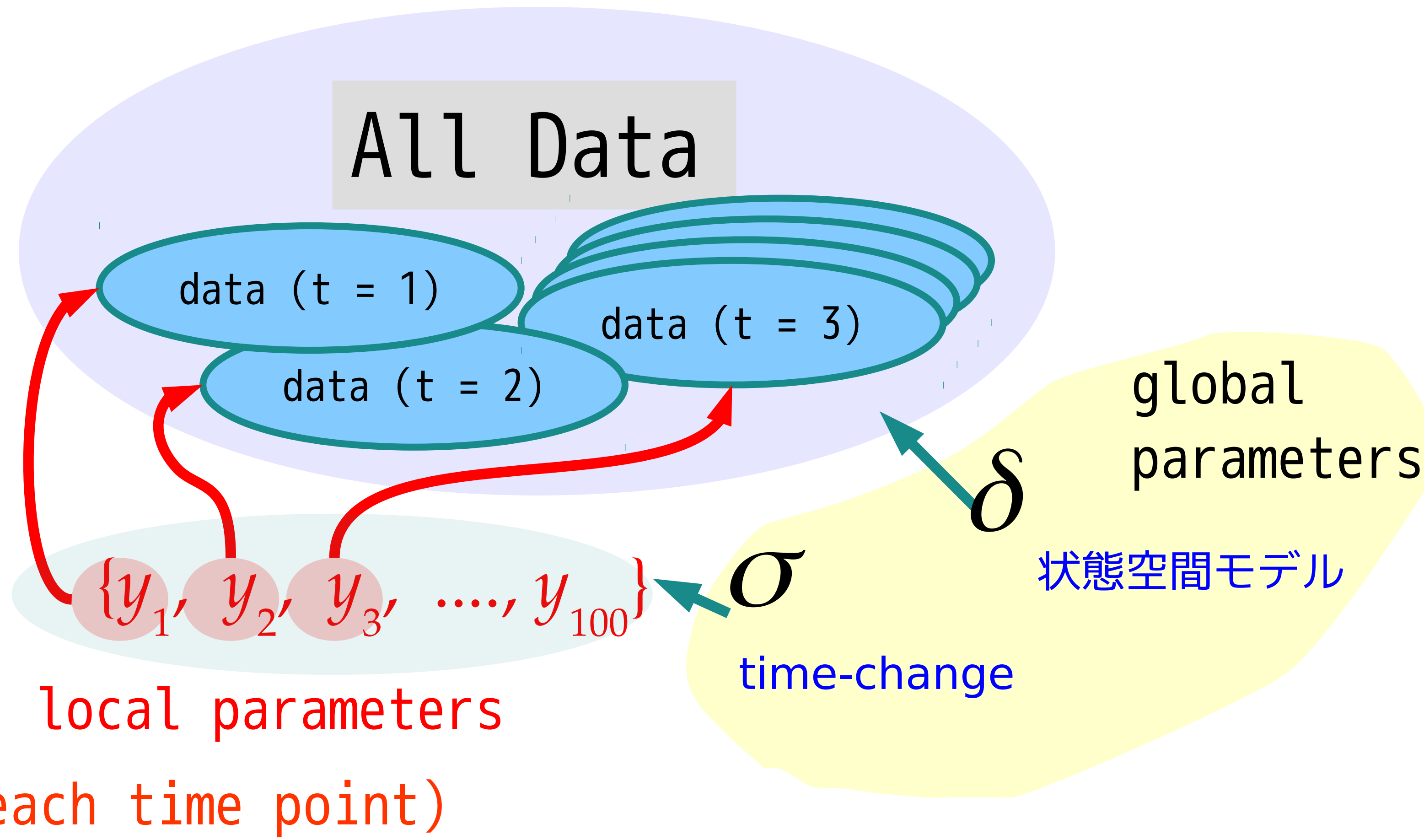


状態空間モデルは階層ベイズモデル

多数の「似たようなパラメーター」たちに
「適切」な制約を加えて推定できる



State space model for TS data, a hiererachical Bayesian model!



状態空間モデルを使う利点

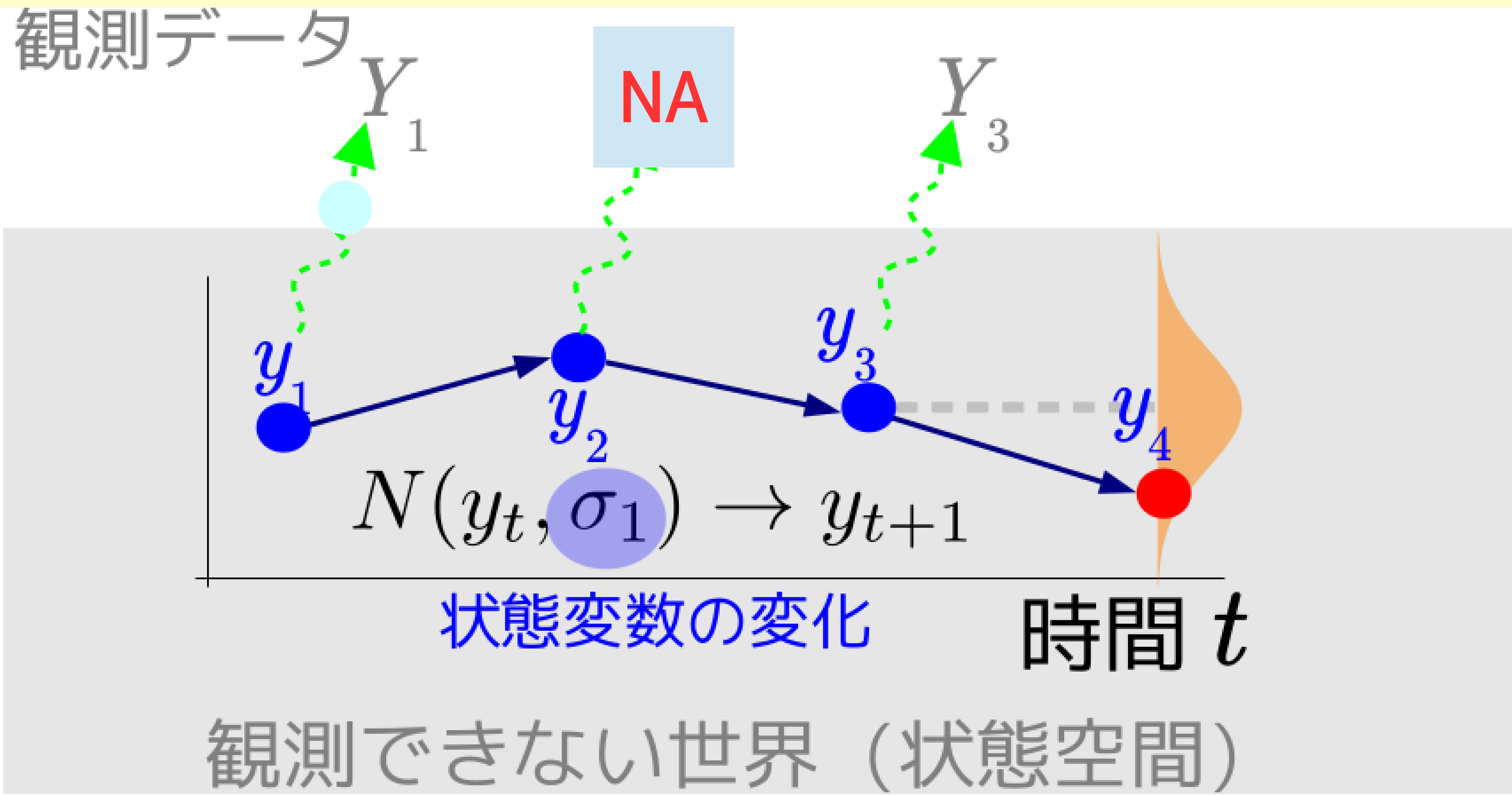
欠測とか不等間隔とか

missing data and
heterogenious time-point

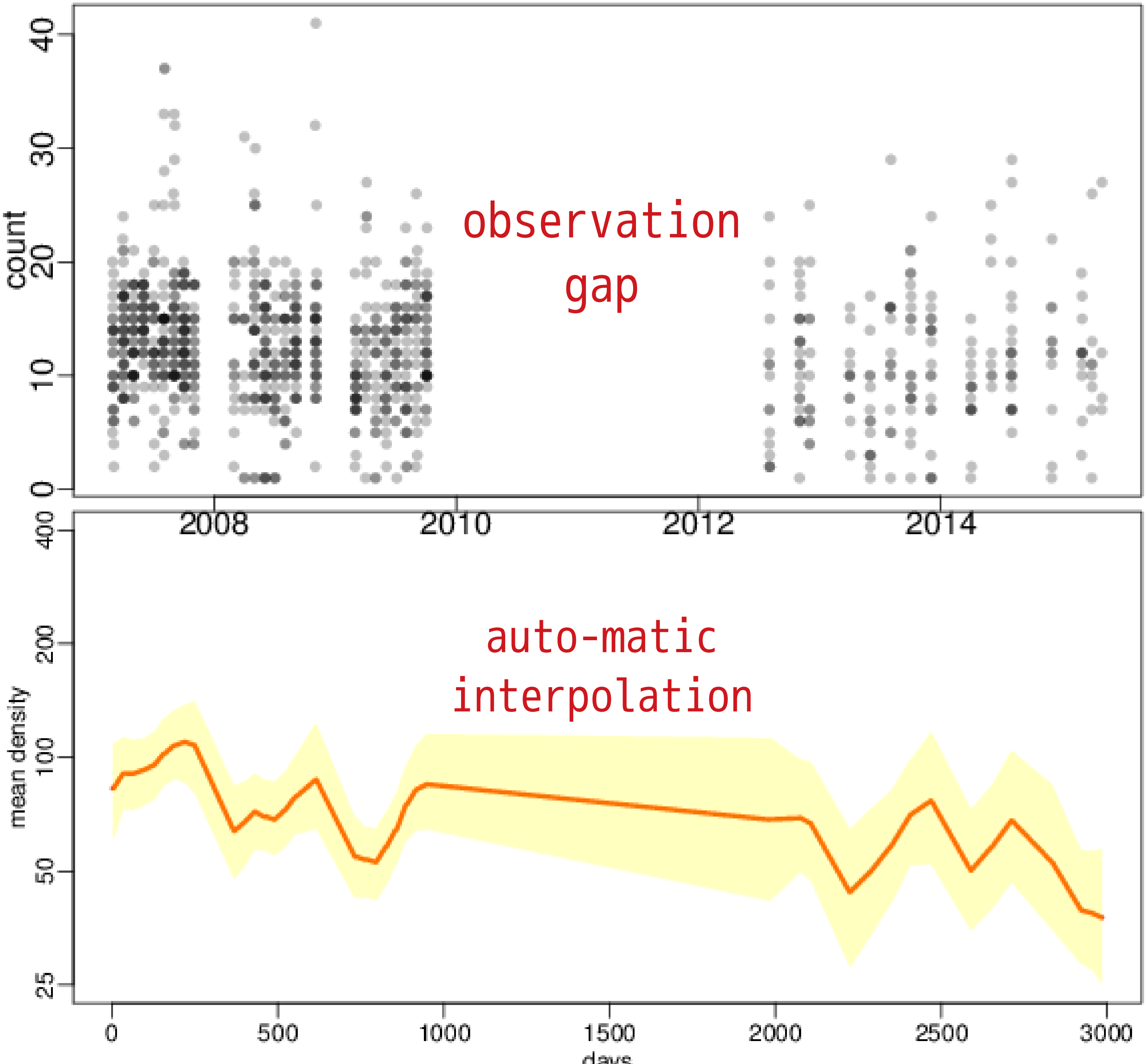
Use state-space model !

状態空間モデル + 観測モデル

欠測があっても問題ない
「補完」の必要なし!



不等間隔データでも何とかなります!



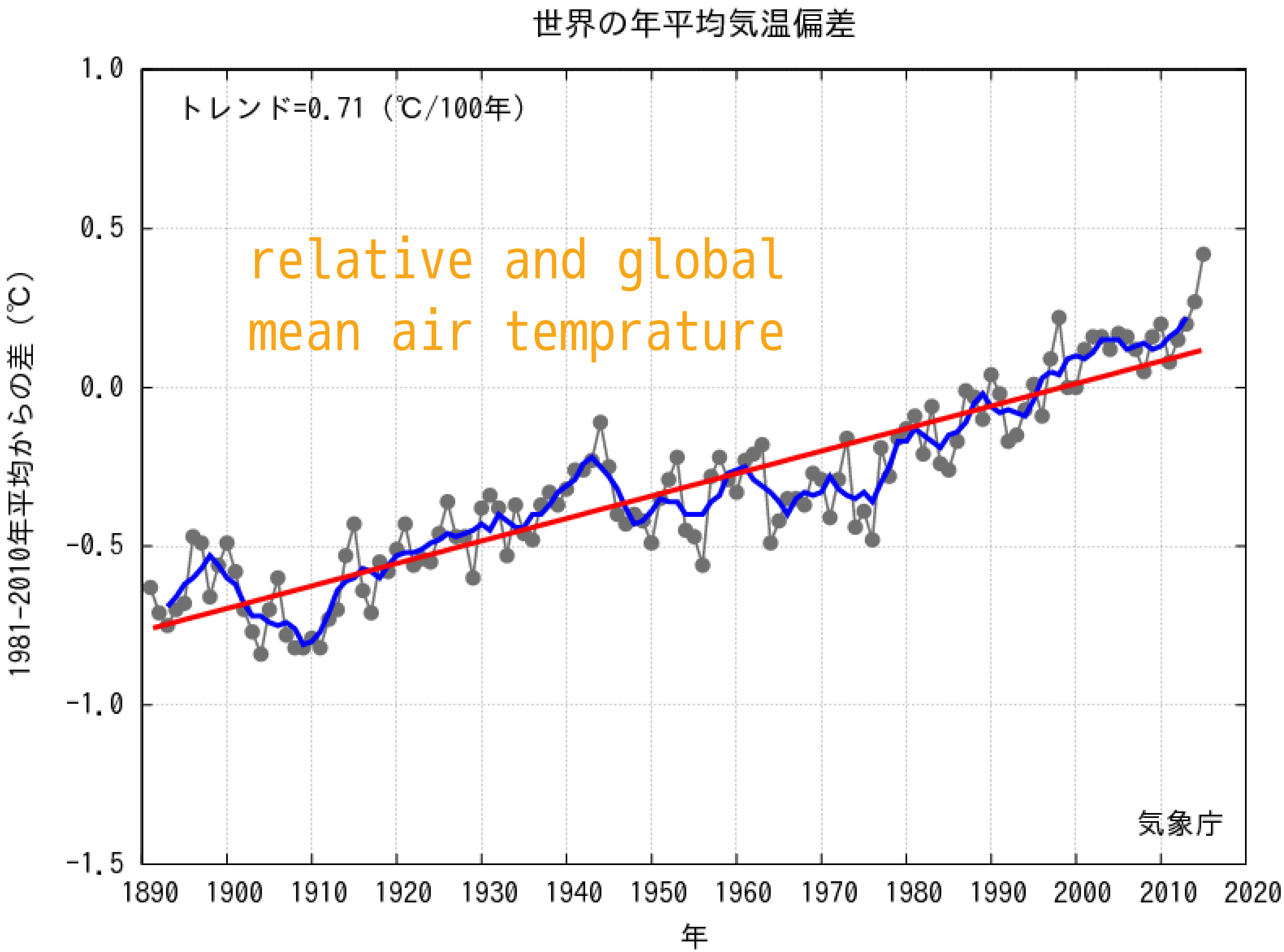
Use State-Space Model!

「ばらばら解析」の回避
気象庁のデータ解析？

An example:

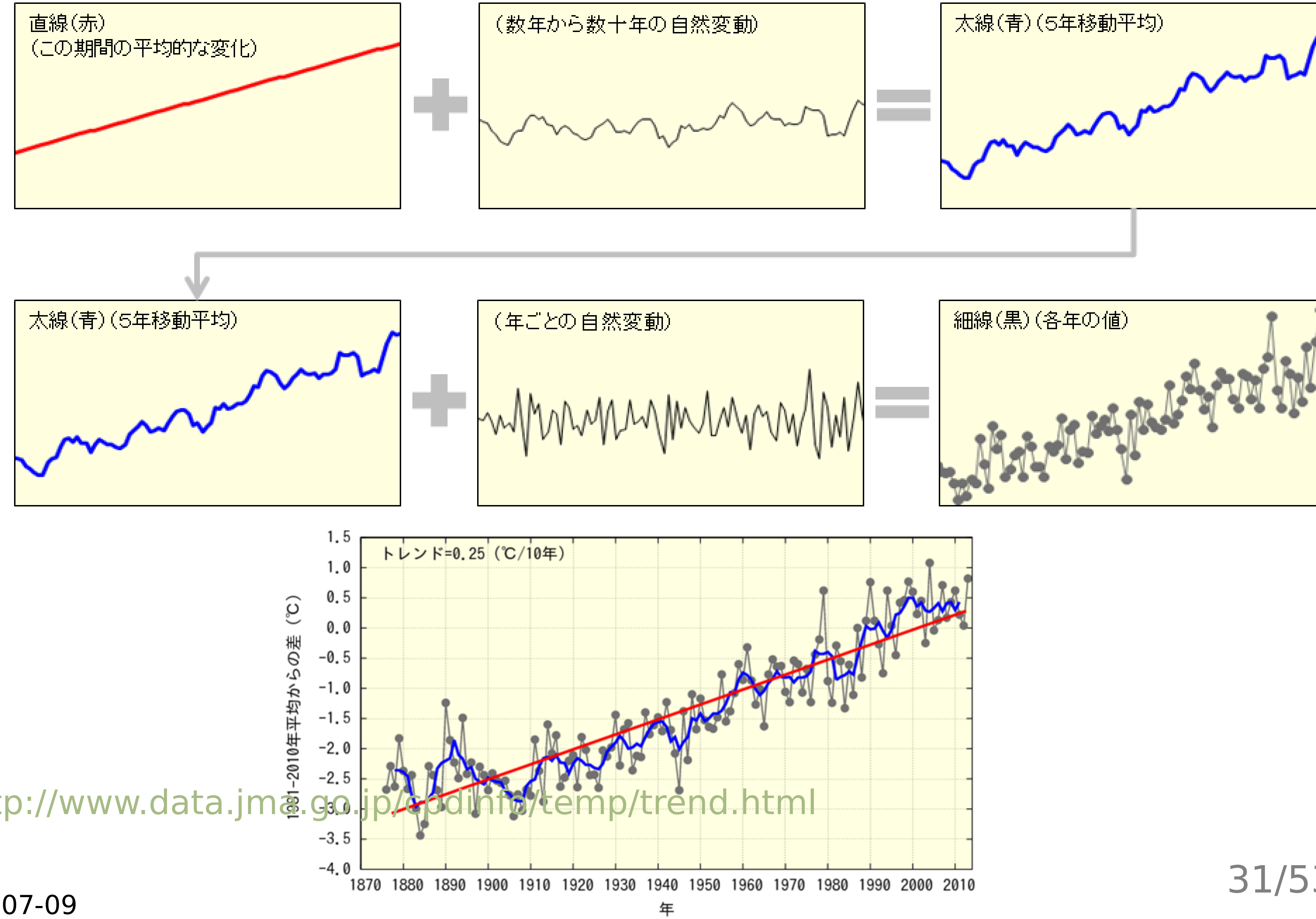
a data set of time series
data: “Is it global warming?”

気象庁の長期変化傾向（トレンド）の解説



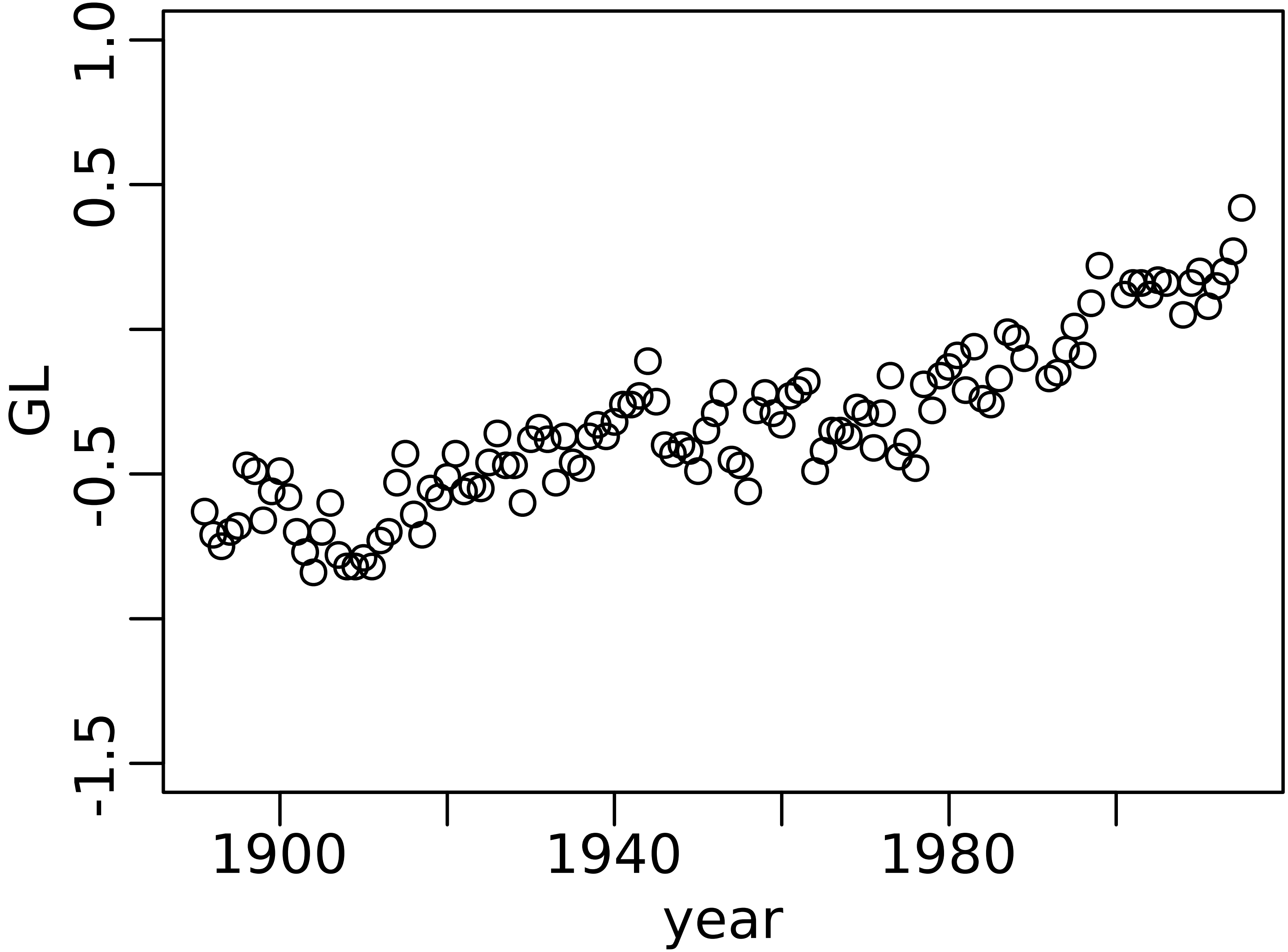
http://www.data.jma.go.jp/cpdinfo/temp/an_wld.html

気象庁の長期変化傾向（トレンド）の解説



http://www.data.jma.go.jp/cpdinfo/temp/trend.html

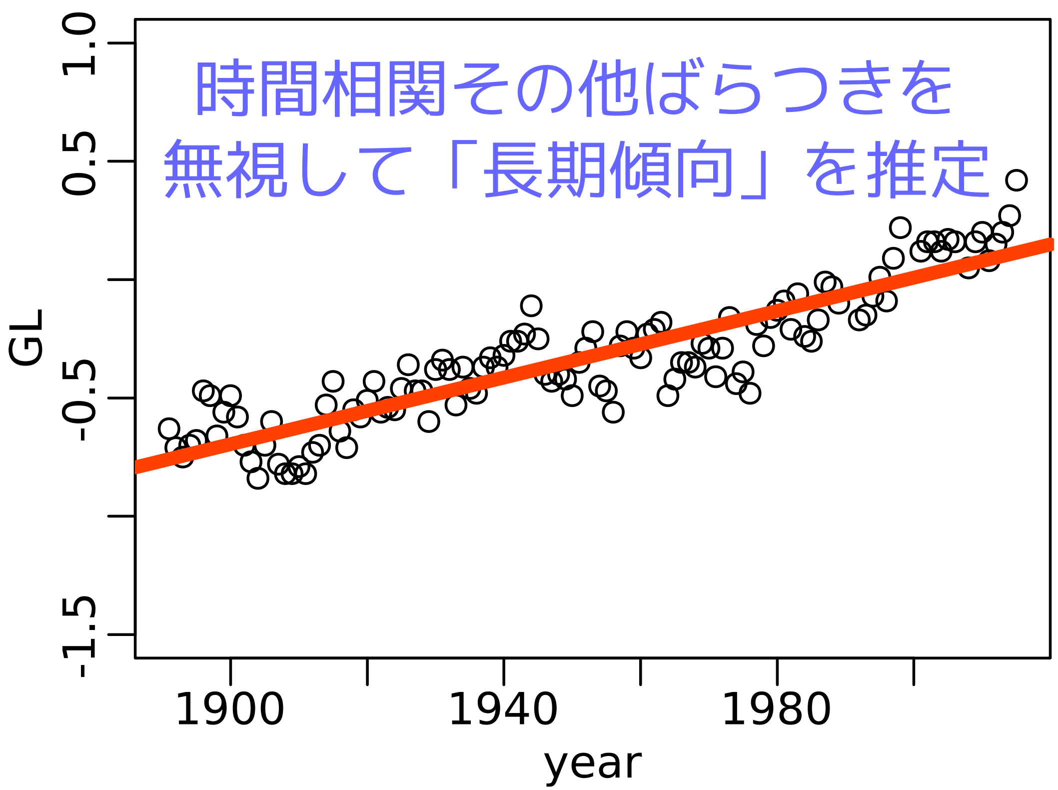
Global warming data



GLM: is it OK? too small p-value!

```
> summary(glm(GL ~ year, data = d))

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.41e+01   6.21e-01   -22.6   <2e-16
year         7.03e-03   3.18e-04    22.1   <2e-16
```

100年あたり 0.70℃

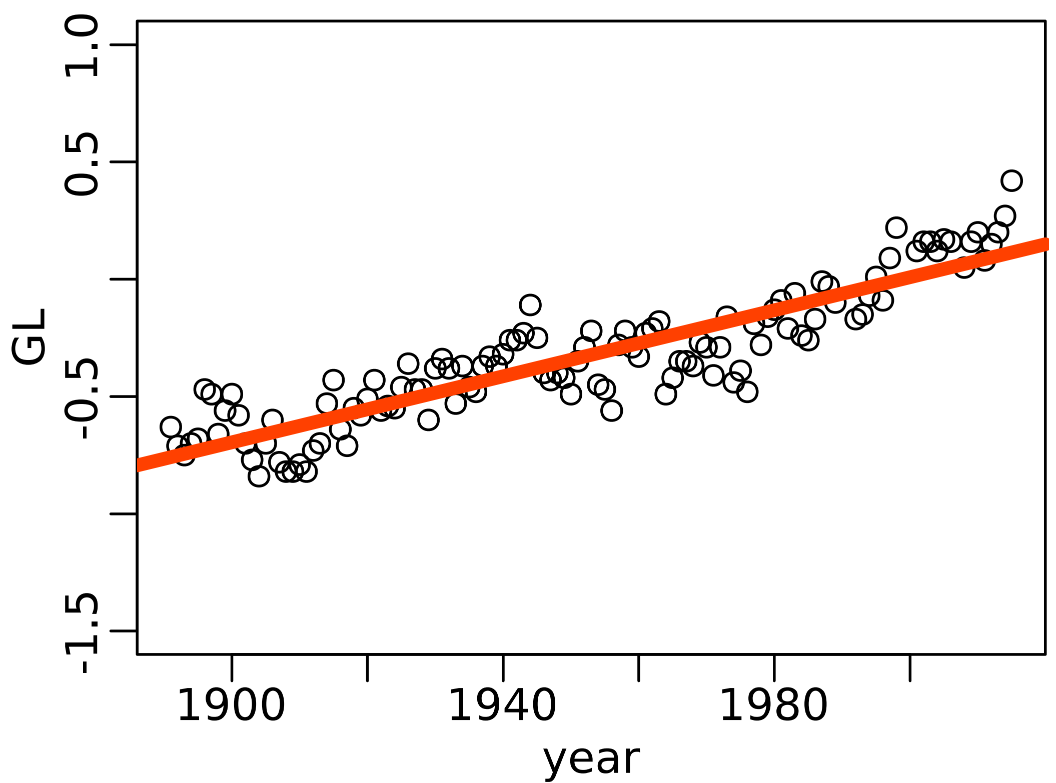


確率 1京ぶんの
2?

直線あてはめ（GLM）が予測した「温暖化」

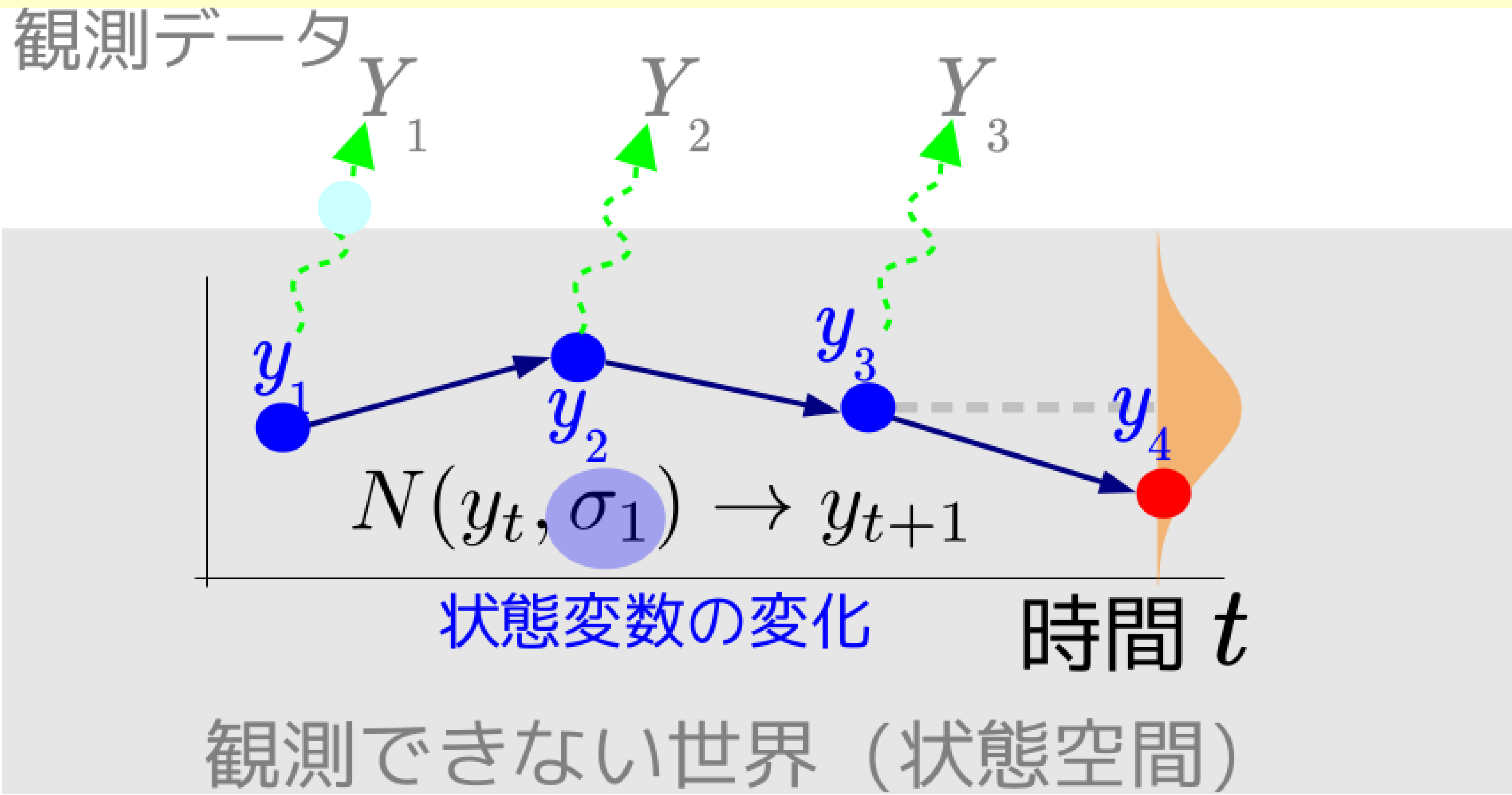
```
> summary(glm(GL ~ year, data = d))

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.41e+01   6.21e-01   -22.6   <2e-16
year         7.03e-03   3.18e-04    22.1   <2e-16
```

100年
あたり
0.70℃



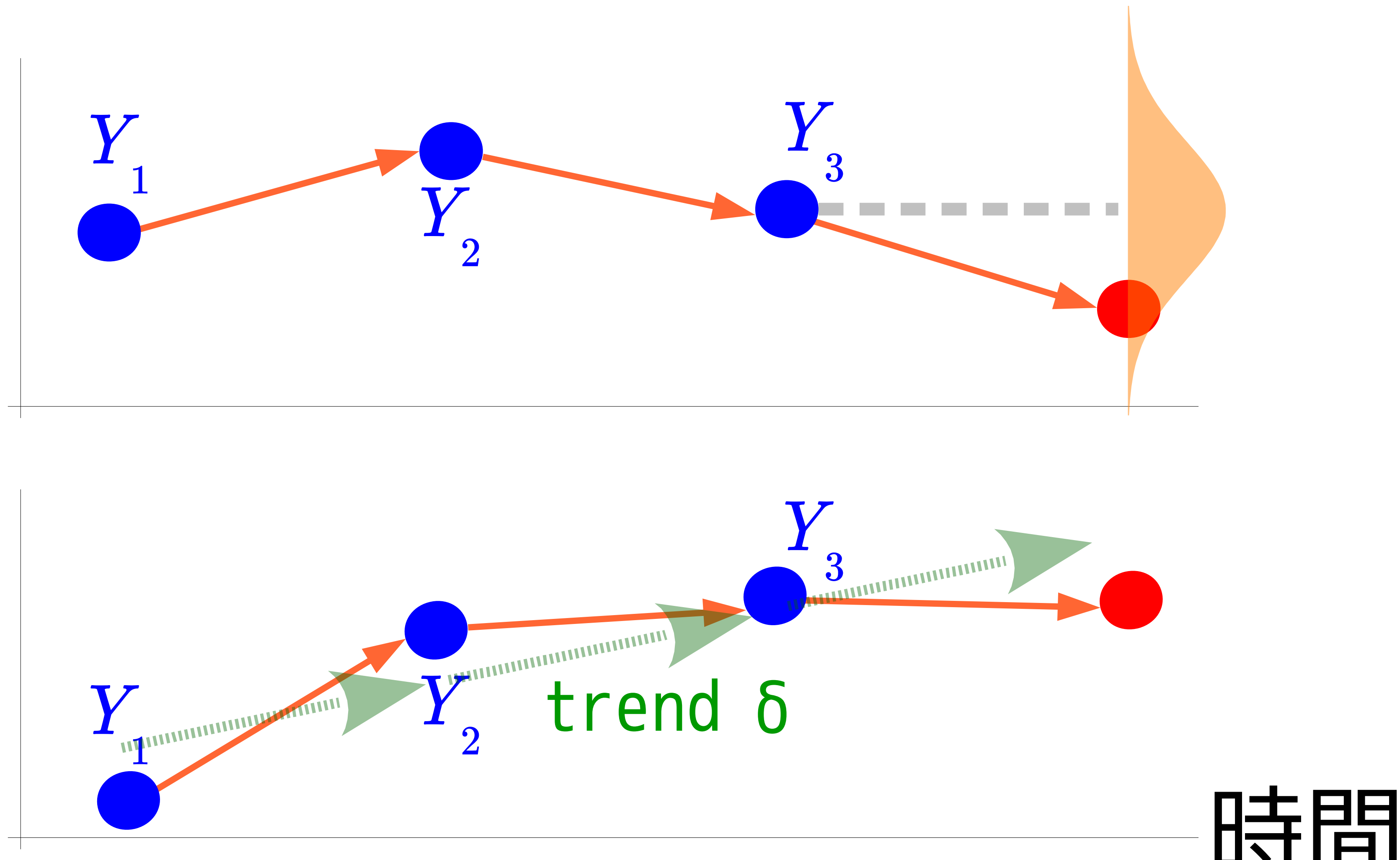
Apply State Space Model (SSM)!

ランダムウォーク+各年独立なノイズ



SSM: Random walk + noise + trend

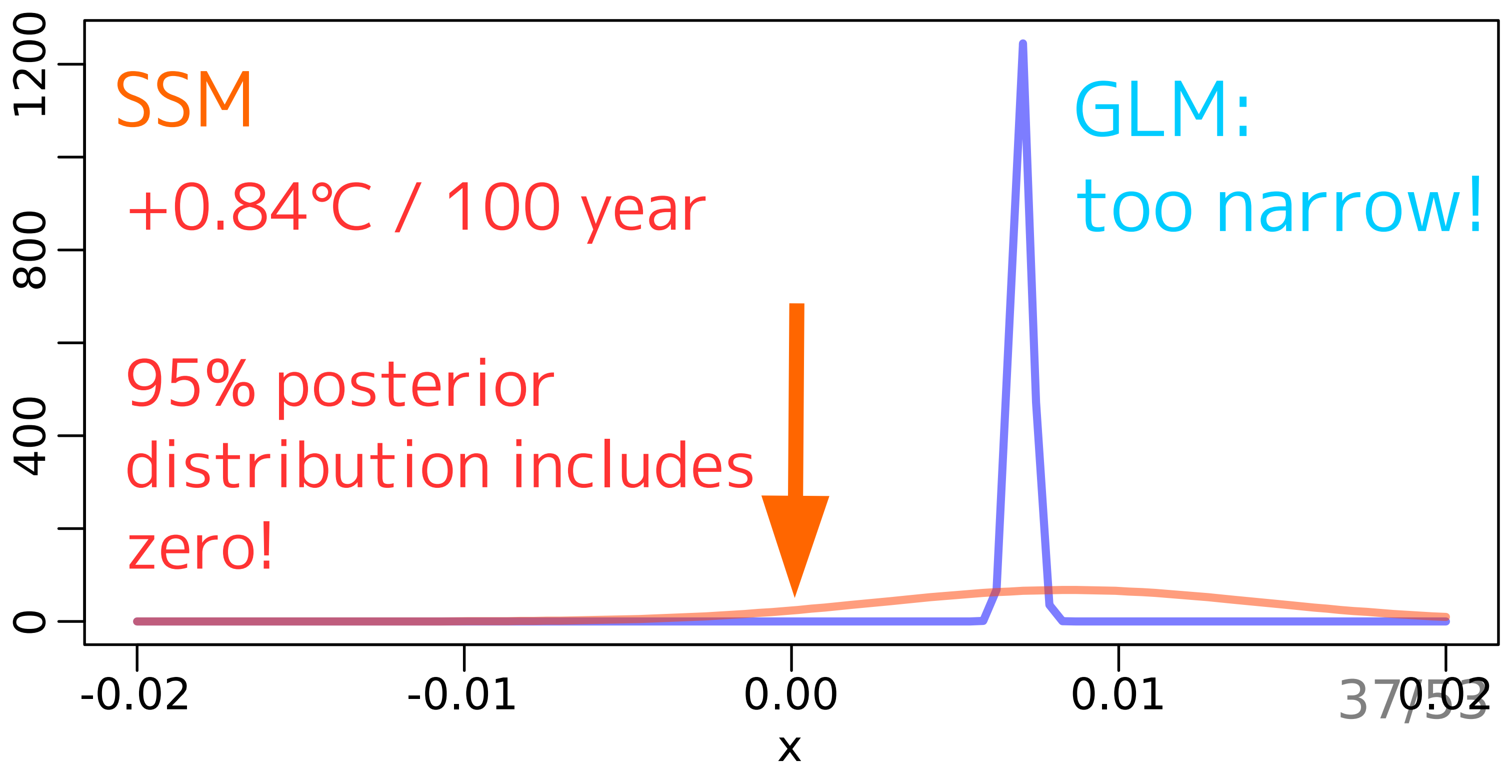
ランダムウォーク+各年独立なノイズ



SSM reveals “uncertainness in global warming”

```
> summary(glm(GL ~ year, data = d))

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.41e+01   6.21e-01   -22.6   <2e-16
year         7.03e-03   3.18e-04    22.1   <2e-16
```

+0.70℃
/ 100 year



疑わしい回帰
spurious regression

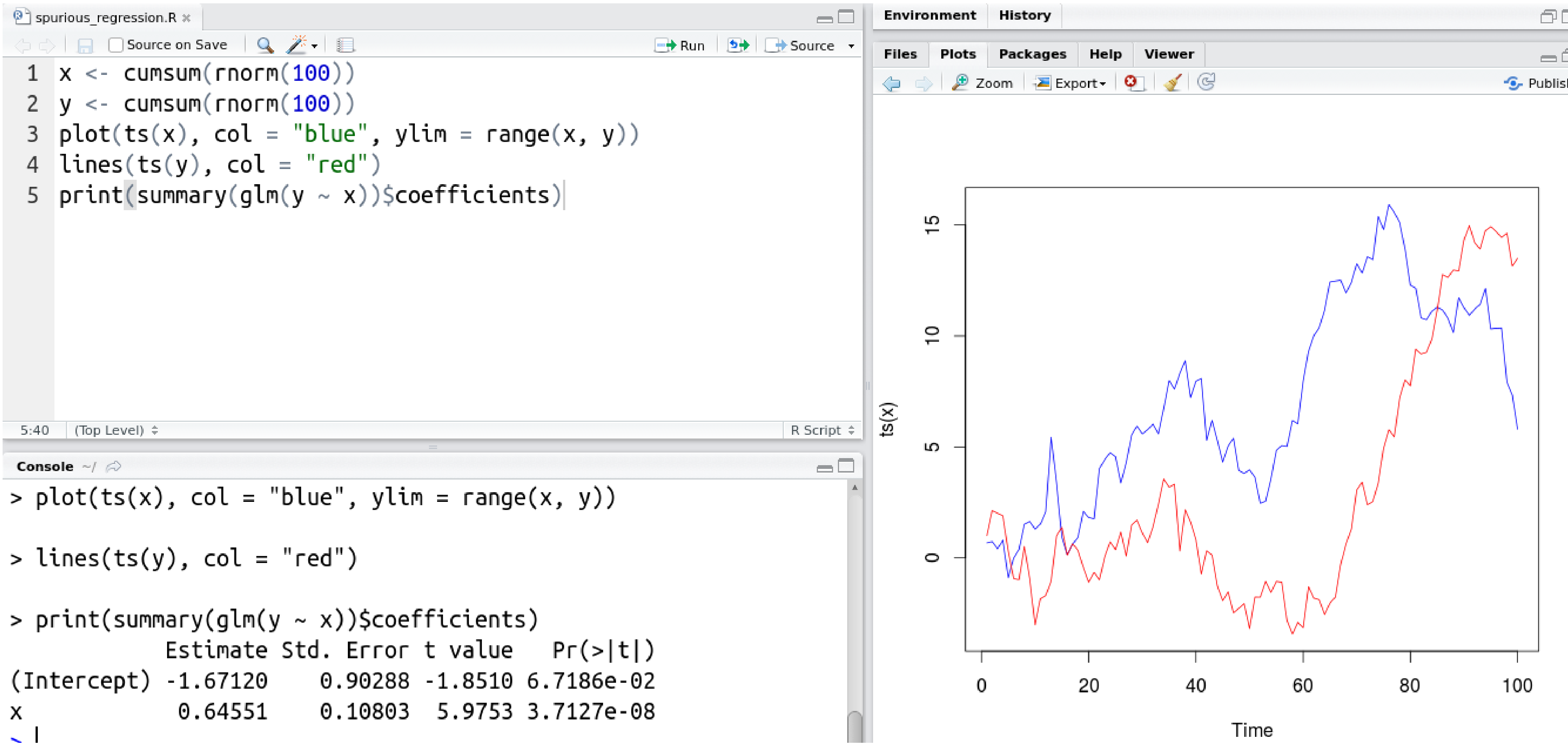
時系列どうしの回帰

time series Y ~ time series X

TS modeling: NOT to do ...

- GLM: Y(t) ~ t and Y(t) ~ X(t)
- combine measurements
- residual analysis
- ... and so on ...

「見せかけの回帰」spurious regression



ちょっとだけ実演してみます

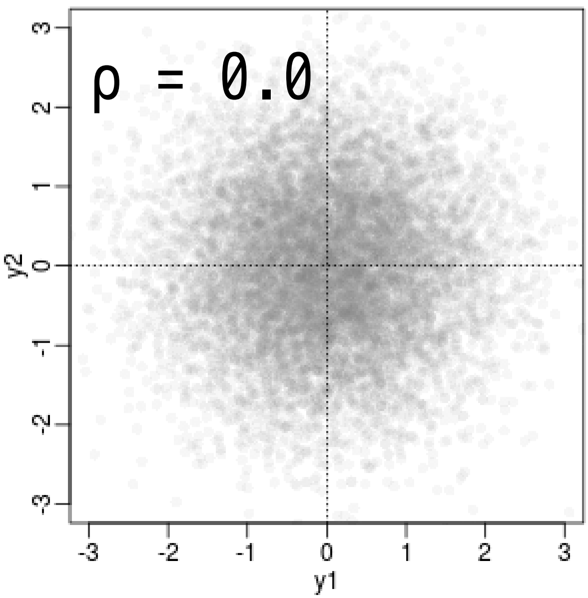
疑わしい回帰
spurious regression

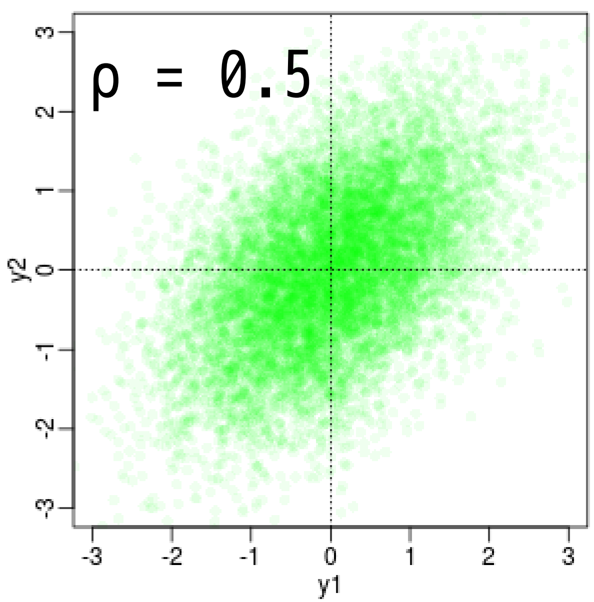
How about fitting

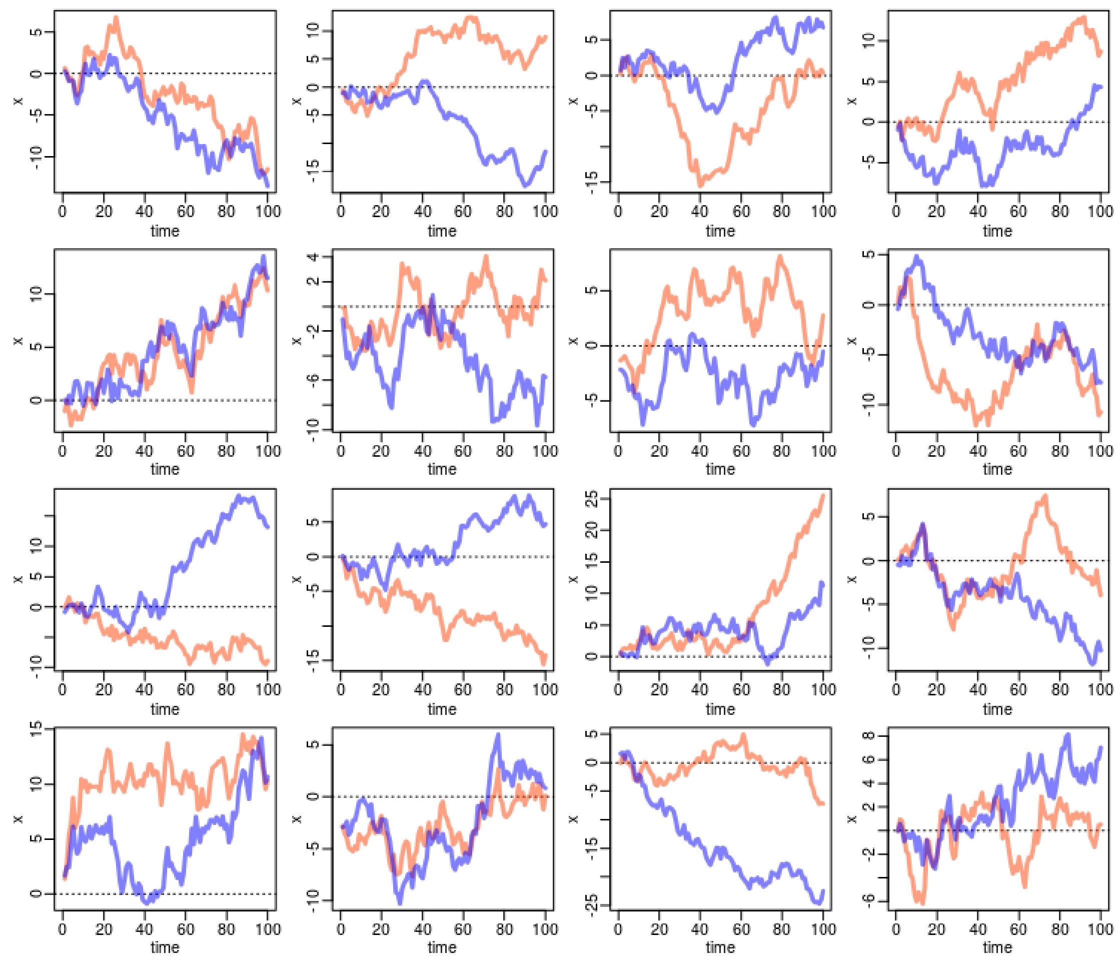
state-space model to estimate

correlations betwee two set of TS

二変量正規分布とランダムウォーク







二変量正規分布を部品とする状態空間モデル

apply bivariate state-space models
including variance-covariance matrix

```
for (i in 1:N.Y) {
    Y[i, 1:2] ~ dmnorm(mu[1:2], Omega[1:2, 1:2])
}
mu[1] ~ dunif(-1.0E+4, 1.0E+4)
mu[2] ~ dunif(-1.0E+4, 1.0E+4)
Omega[1:2, 1:2] <- inverse(VarCov[1:2, 1:2])
VarCov[1, 1] <- sigma[1] * sigma[1]
VarCov[1, 2] <- sigma[1] * sigma[2] * rho
VarCov[2, 1] <- sigma[2] * sigma[1] * rho
VarCov[2, 2] <- sigma[2] * sigma[2]
sigma[1] ~ dunif(0.0, 1.0E+4)
sigma[2] ~ dunif(0.0, 1.0E+4)
rho ~ dunif(-1.0, 1.0)
```

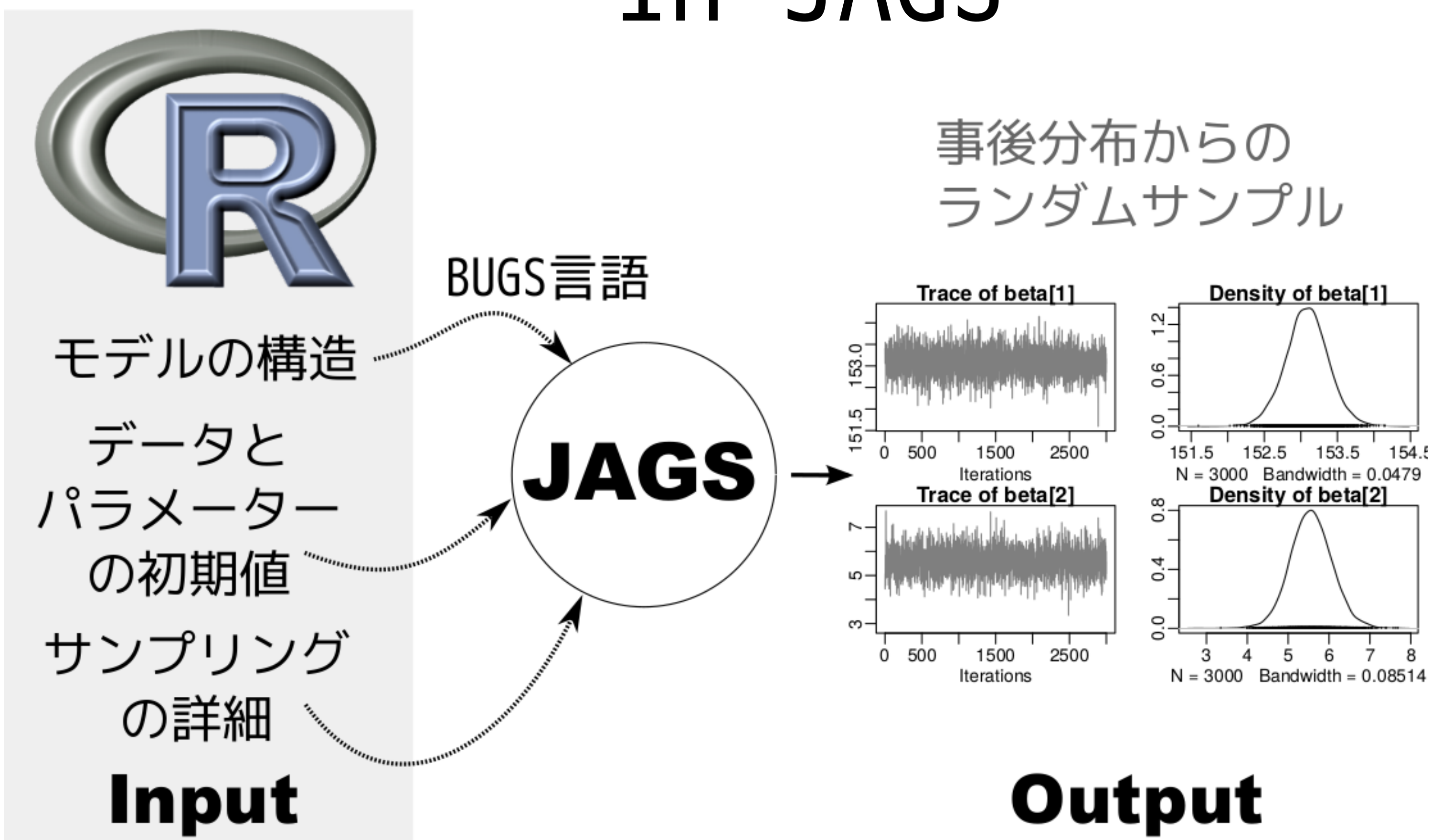
bivariate state space model estimates the posterior of variance and covariance matrix

```
 3 chains, each with 5200 iterations (first 200 discarded)
 n.sims = 15000 iterations saved
           mean    sd   2.5%    25%    50%    75% 97.5%  Rhat n.eff
mu[1]    -0.122 0.110 -0.342 -0.195 -0.120 -0.048 0.090 1.001  6000
mu[2]    -0.157 0.100 -0.355 -0.224 -0.157 -0.091 0.041 1.002  1500
sigma[1]  1.091 0.079  0.949  1.036  1.086  1.142 1.261 1.001  6100
sigma[2]  0.993 0.074  0.864  0.941  0.987  1.039 1.151 1.001  4100
rho       0.568 0.070  0.420  0.523  0.573  0.617 0.693 1.001 11000
```

ふたつの時系列データの変動が
相関しているかどうかを特定できる

MCMC parameter estimation

Hierarchical model written in JAGS



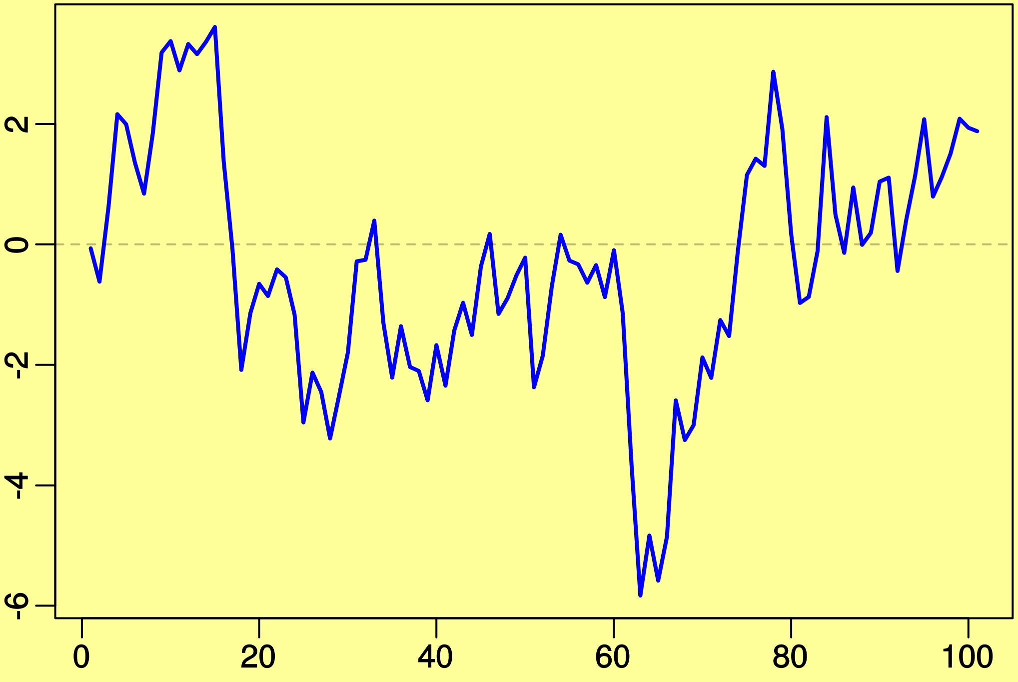
```
model
{
    Tau.Noninformative <- 0.0001
    Y[1] ~ dnorm(y[1], tau[2])
    y[1] ~ dnorm(0, Tau.Noninformative)
    for (t in 2:N.Y) {
        Y[t] ~ dnorm(y[t], tau[2])
        y[t] ~ dnorm(m[t], tau[1])
        m[t] <- delta + y[t - 1]
    }
    delta ~ dnorm(0, Tau.Noninformative)
    for (k in 1:2) {
        tau[k] <- 1 / (s[k] * s[k])
        s[k] ~ dunif(0, 10000)
    }
}
```

Apply hierarchical models
to time-series (TS) data!
i.e. State Space Models

σ_2 小

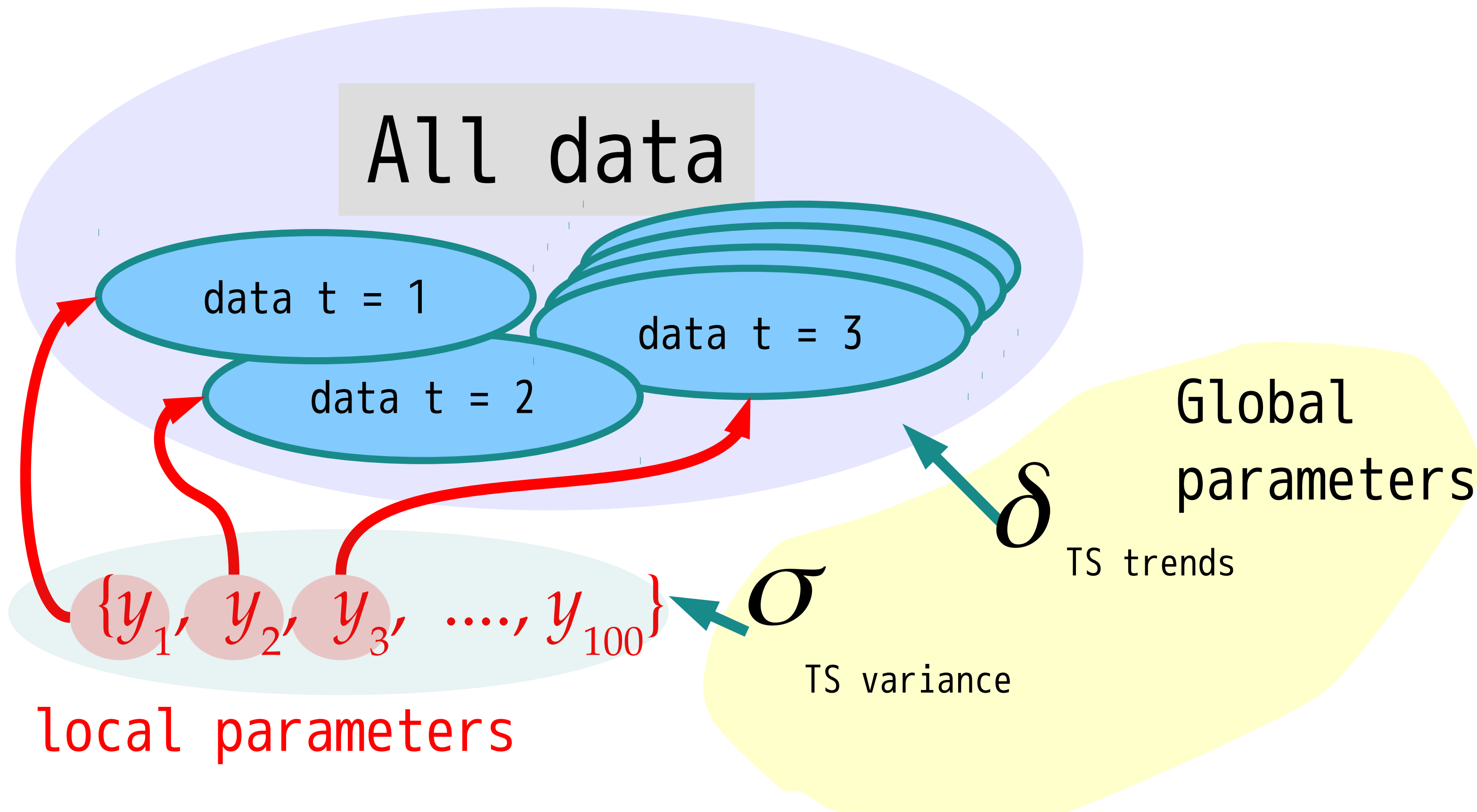
σ_1 大

$\delta = 0$



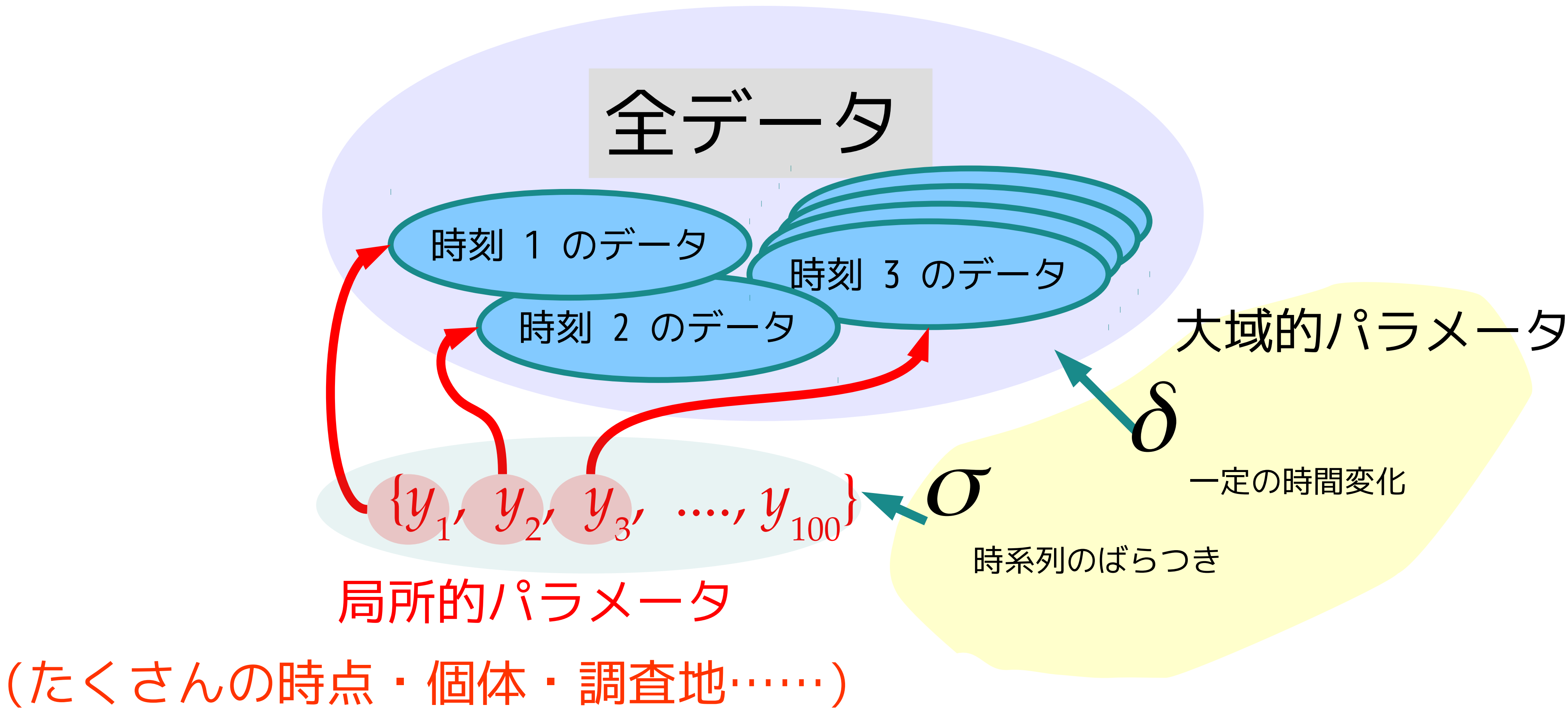
Hierarchical Model is powerful!

While GLM can not model TS data,
Hierearchical model is effective!



階層ベイズモデルとは?

多数の「似たようなパラメーター」たちに
「適切」な制約を加えて推定できる



How do you fit TS model to data?

R packages to estimate

parameters in stat-space models

```
library(dlm)

library(KFAS)
```

Also we have more powerful

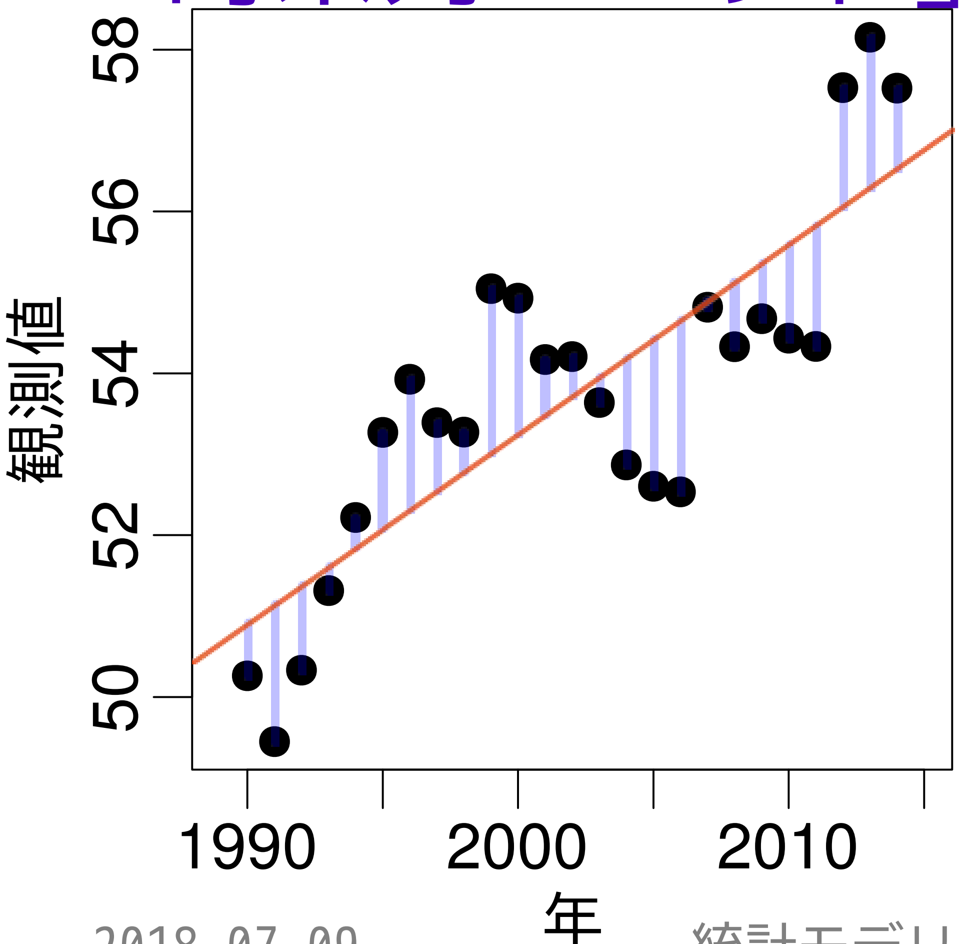
models to analyze TS data...

おわりに

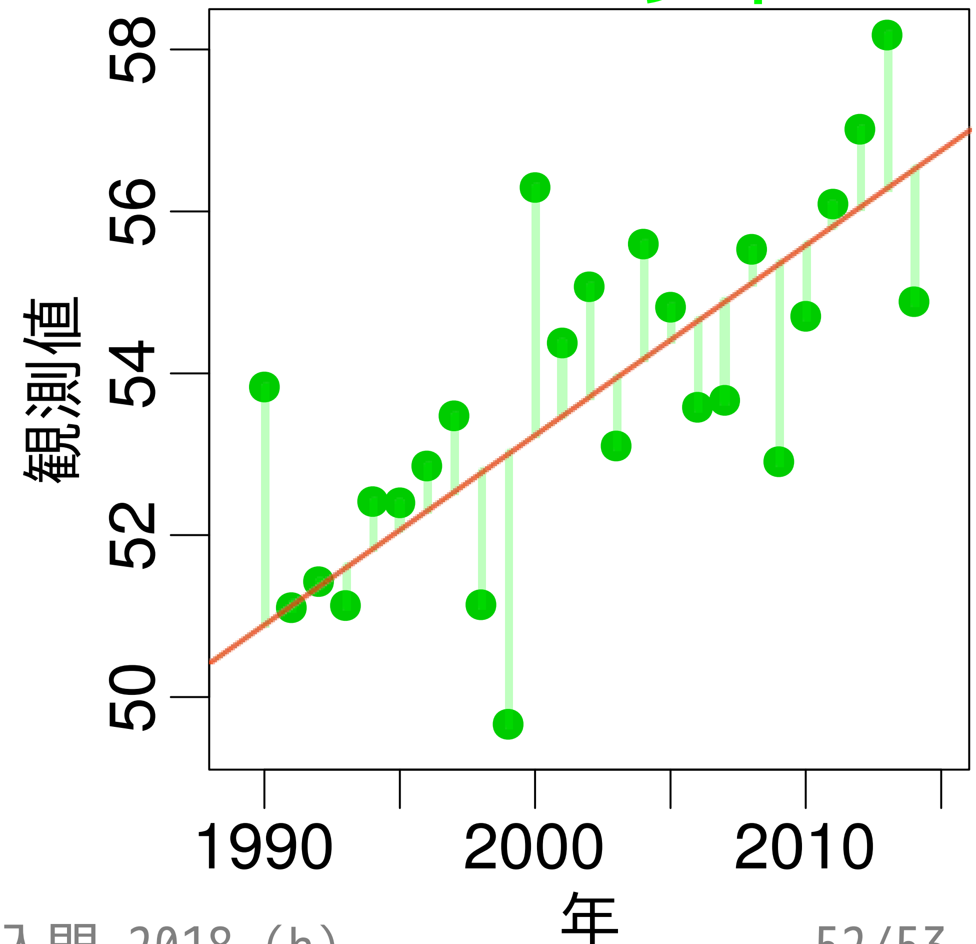
take-out message

Modeling time series data can be a hierarchical modeling

時系列の「ずれ」



GLM のずれ



The End: have a nice statistical modeling!

The Evolution of Linear Models

Parameter Estimation

Hierarchical Bayesian Model (HBM) — MCMC

Generalized Linear Mixed Model (GLMM)

Generalized Linear Model (GLM) — MLE

Linear Model — MSE

データ解析は

階層ベイズモデルで！